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HELP, A MULTI-MATERIAL EULERIAN PROGRAM FOR COMPRESSIBLE FLUID

AND ELASTIC-PLASTIC FLOWS IN TWO SPACE DIMENSIONS AND TIME

VOLUME I

Prepared by

Systems, Science and Software
La Jolla, California

May 1971

U.S. ARMY ABERDEEN RESEARCH AND DEVELOPMENT CENTER
BALLISTIC RESEARCH LABORATORIES
ABERDEEN PROVING GROUND, MARYLAND

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LJHageman/JMWalsh
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ABSTRACT

Volume I details a new numerical model for solving multi-material
flows which are functions of two space dimensions and time. The program,
called HELP, is basically Eulerian but also contains Lagrangian features
for the explicit definition of interface positions. The program is gen-
eral in the sense that any number of materials can be present in a given
Eulerian cell and no special difficulties arise for flows involving
extreme material distortions.

The program is suitable for compressible media with strength depen-
dence. Compressibility is included by an equation of state giving pres-
sure as a function of density and specific internal energy. Strength is
included by means of a constitutive equation giving the deviatoric
stresses as functions of the elastic and plastic strains.

In this Volume I report, the basic equations are developed, the
computer program is documented and sample results are given from six
applications.

CONTENTS

Volume I

Contents (continued)

Contents (continued)

Contents (continued)

Volume II of this two-volume set contains the
FORTRAN listing of HELP.

I. INTRODUCTION

HELP is an attempt to address and resolve certain familiar and long-standing problems in hydrodynamic computing which are not adequately resolved by pure Lagrangian or Eulerian methods. Describing the motion of a material interface in a Lagrangian code is automatic, but the numerical solution becomes unsatisfactory if the material undergoes extreme distortion. In Eulerian codes, on the other hand, extreme distortion poses no problem, but the accurate treatment of surface motion (i.e., a material interface or a free surface) requires a detailed knowledge of surface location which is not contained in a purely Eulerian program. In the present work, massless tracer particles are introduced which define the surface positions and move across the Eulerian grid, so that the program yields a Lagrangian-type definition of the moving surfaces. At the same time the code does not lose the Eulerian capability for treating extreme distortions without difficulty. Transport from Eulerian cells containing either free surfaces or interfaces between two or more materials is done in a manner which takes account of these surface positions.

The HELP code has evolved from four major hydrodynamic programs developed over the past fifteen years. These codes have been used in a wide range of applied problems in shock hydrodynamics. Starting from the one-material code RPM[1] the HELP development reported here adds the capabilities of describing multi-material interactions and treats material strength as elastic-plastic* rather than rigid-plastic. RPM, in turn, was an extension of the OIL compressible fluid code[2]

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*The name HELP means Hydrodynamic plus ELastic Plastic.

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to include material strength in the rigid-perfectly plastic approximation. OIL in turn evolved from the PIC[3] code and was developed by replacing the discrete PIC particles by a continuum. This latter transition to a continuum calculation was made in order to permit the treatment of very small compressions (such as those usually accompanying processes in which material strength is important), without having to use an extremely large number of particles for adequate mass resolution. A disadvantage of this transition was the loss of certain Lagrangian-type features of PIC, and specifically the ability to treat flows containing more than one material. The major purpose of the work on HELP (other than the effort to treat strength with elastic and plastic effects) has been the restoration of this multi-material capability, while retaining the ease of treating small compressions and the computing economy of a continuous Eulerian model.

In all of the programs mentioned above, the basic difference equations are explicit and are accurate to terms of first order. The difference equations used in HELP will be given in this report, but the interested reader is referred also to Refs. 1 through 3, and literature cited therein, for further discussions of the properties of these or very similar difference equations.

The present code development has successfully led to the ability to treat extreme strength-dependent deformations in multi-material interactions. Examples are the axisymmetric interaction of projectiles of various shapes with target plates of different materials.[4] Similarly, in Section VI of this report are seen results of calculations from a half-dozen representative problems. It is still possible with the program to treat the simpler cases of the earlier codes; specifically, strength effects can be ignored by simply bypassing one phase of the calculation, and no changes are required to apply the program to flow fields containing a single material.

HELP cannot be utilized effectively without investing
a considerable effort, perhaps 1 to 2 manpower for several
months, in becoming familiar with the basic numerical method,
the details of the FORTRAN program, and in gaining computing
experience with the code. While every effort has been made
to make the code as reliable and general as possible, it
should be noted that any new application may give rise to
problems or anomolous results (e.g., a failure of the pressure
iteration in a mixed cell) which can effectively be diagnosed
and resolved only by persons who are sufficiently familiar
with the entire program that they can rapidly identify and
remedy difficulties with a calculation. Some such general
statements are, of course, true of any 2-D hydrodynamics code
as applied to a new problem. In the case of HELP, working
familiarity with one of the predecessor codes RPM, OIL or PIC
will be of considerable value in reducing the above learning
period. In the present report, an attempt is made to give
the potential user a comprehensive introduction to both the
numerical methods and the computer program.

Some comments will guide the reader through the text
of this report. The next section (II) is a general descrip-
tion of the numerical method. In Section III, the treatment
of strength with the elastic-plastic constitutive equation,
is reported in some detail because these results are essen-
tially new. Similarly, Section IV describes in some detail
the method which is used for propagating interfaces and free
surfaces, and the manner in which information on interface
position is used in affecting the Eulerian hydrodynamic solution.
Section V details the pressure iteration which is necessary
in order to determine the pressure (and material densities)
in cells containing more than one material. Section VI pre-
sents sample results obtained with HELP. Section VII is a
guide to the FORTRAN program, giving such information as the

flow diagram, the functions of the various subroutines, the input for generating and restarting problems, and the dictionary of FORTRAN variables. Volume II contains a listing of the FORTRAN program. The program is interspersed with comment cards in an attempt to facilitate learning.

Learners, users and improvers of HELP are invited to communicate with the authors where such improvement might facilitate their efforts or lead to improvements in the program.

II. GENERAL DESCRIPTION OF THE NUMERICAL METHOD

In the present section we give the conservation equations and the numerical model which is used to treat these equations. The equation of state which is used for condensed media is also presented with equation-of-state parameters for some nineteen materials.

2.1 CONSERVATION EQUATIONS

Space is divided into fixed Eulerian cells through which the fluid moves. To arrive at expressions for the rate of change of total mass, momentum and energy within such a cell, it is convenient to start with the equations of motion in the form:

$$\frac{\partial \rho}{\partial t} = - \frac{\partial}{\partial x_i} \left(\rho u_i \right) \tag{1}$$

$$\rho \frac{Du_j}{Dt} = \frac{\partial}{\partial x_i} \left(\sigma_{ij} \right) \tag{2}$$

$$\rho \frac{DE_T}{Dt} = \frac{\partial}{\partial x_i} \left(\sigma_{ij} u_j \right) \tag{3}$$

Here σ_{ij} is the stress tensor, which can be regarded as the sum of the hydrostatic stress, $-\delta_{ij}P$, and a stress deviator tensor, s_{ij}, i.e.,

$$\sigma_{ij} = s_{ij} - \delta_{ij}P \tag{4}$$

and E_T is the total energy (kinetic plus internal) per unit mass. Tensor notation is implied, so that repeated indices denote summations.

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Expanding the convective derivatives in Eqs. (2) and (3), $Df/Dt = \partial f/\partial t + u_i \, \partial f/\partial x_i$, then adding Eq. (1) times u_j to Eq. (2), and Eq. (1) times E_T to Eq. (3), and collecting terms, gives

$$\frac{\partial}{\partial t}\left(\rho u_j\right) = \frac{\partial}{\partial x_i}\,\sigma_{ij} - \frac{\partial}{\partial x_i}\left(\rho u_i u_j\right) \tag{5}$$

$$\frac{\partial}{\partial t}\left(\rho E_T\right) = \frac{\partial}{\partial x_i}\left(\sigma_{ij}u_j\right) - \frac{\partial}{\partial x_i}\left(\rho u_i E_T\right). \tag{6}$$

For the developments to follow it is desirable to replace these differential equations by the analogous integral equations, obtained by integrating over the cell volume, V, and then converting the volume integral of divergences to surface integrals over the cell surfaces. Equations (1), (5), and (6) then become

$$\frac{\partial}{\partial t}\int_V \rho \, dV = -\int_s \rho u_i n_i \, ds \tag{7}$$

$$\frac{\partial}{\partial t}\int_V \rho u_j \, dV = \int_s \sigma_{ij} n_i \, ds - \int_s \rho u_i u_j n_i \, ds \tag{8}$$

$$\frac{\partial}{\partial t}\int_V \rho E_T \, dV = \int_s \sigma_{ij} u_j n_i \, ds - \int_s \rho u_i E_T n_i \, ds \,. \tag{9}$$

2.2 DIFFERENCE EQUATIONS

2.2.1 Division into Phases

It is convenient to express the integral conservation relations, Eqs. (7) through (9), as finite difference equations over the time step Δt and also to decompose the total stress σ_{ij} into its deviator and hydrostatic components, according to Eq. (4). This gives, for the increments of total mass (m), momenta $\left(mu_j\right)$ and energy $\left(mE_T\right)$ within the cell

$$\Delta m = \qquad\qquad\qquad - \Delta t \int_s \rho u_i n_i ds \tag{10}$$

$$\Delta\left(mu_j\right) = -\Delta t \int_s Pn_j ds + \Delta t \int_s s_{ij}n_i ds \quad - \Delta t \int_s \left(\rho u_i u_j\right)n_i ds \tag{11}$$

$$\Delta\left(mE_T\right) = -\Delta t \int_s Pu_i n_i ds + \Delta t \int_s s_{ij}u_j n_i ds - \Delta t \int_s \left(\rho u_i E_T\right)n_i ds \tag{12}$$

Here, the terms on the right are divided into increments due to pressure forces on the cell surface (first column), those due to the stress deviator forces on the cell surface (second column) and the increments (third column) due to the transports of mass, momentum or energy through the surface of the cell. These three types of contributions are accounted for in the computation in distinct phases. Specifically, during each time step all cells are updated for:

> Pressure effects in Phase 1
> Effects of the stress deviators in Phase 3
> Transport effects in Phase 2

(The calculations are performed in the order listed, naming of the phases having to do with the history of their development.) In the discussion that follows, the calculation of the terms on the right of Eqs. (10) through (12) are described sequentially starting with Phase 1.

Some preliminary definitions will be useful. Superscript n on a variable refers to the value of the variable at the beginning of the time step and superscript (n+1) denotes the value at the end. In this discussion a typical cell in the interior of the grid is considered; Section 2.4.1 has a discussion of the special conditions which describe the calculation at the grid boundaries or at the axis of symmetry. For a typical cell, denoted by a value of the index k, the dependent variables for that cell are written $P(k)$, $u(k)$, $v(k)$, $E_m(k)$, $m(k)$, representing respectively the pressure, radial and axial components of velocity, the specific internal energy and the mass for cell k. The adjacent cells above, below, to the right and left of k will be designated respectively as ka, kb, kr, and kl. Here the terms above, below, right and left refer to a cross-section view of the cells, in which the left border is parallel to the axis of symmetry with z increasing upward (see Fig. 1). Each cell is, then, the torus obtained by rotating the rectangle (since $\Delta r \neq \Delta z$ in general) about the axis of symmetry.

2.2.2 Phase 1—The Effects of Pressure

In the following discussion, the Phase 1 calculation is described first for one-material cells. The changes made to extend the Phase 1 calculation to a multi-material cell are then given in a concluding paragraph.
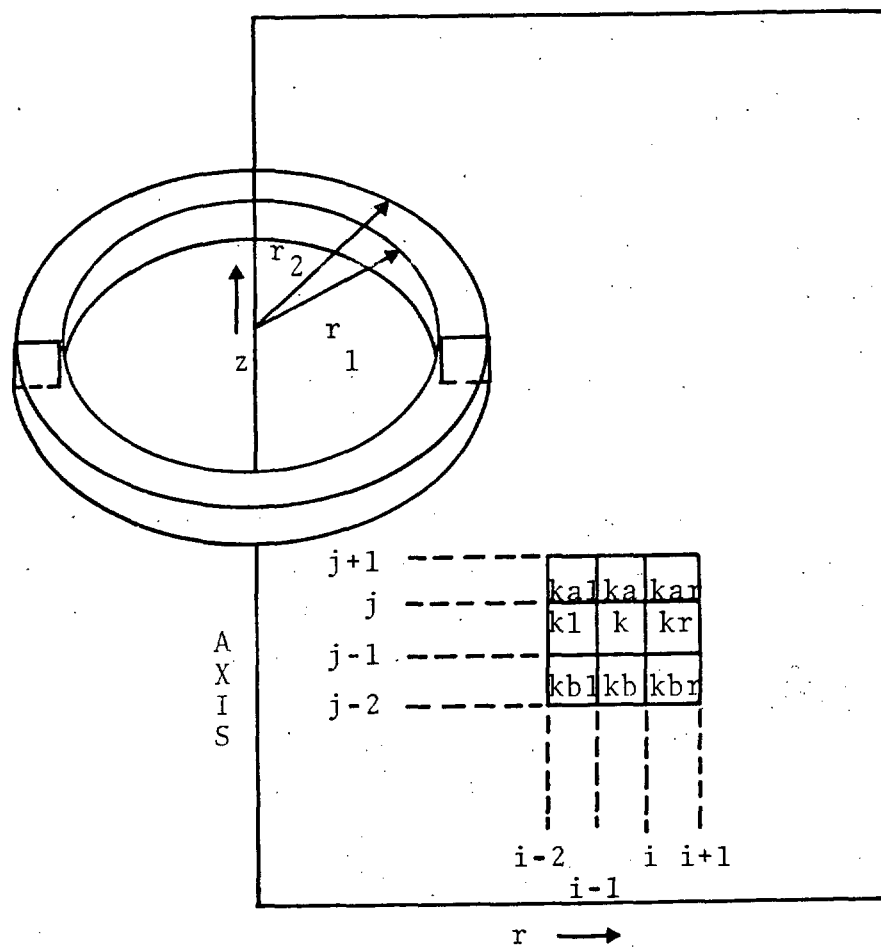
Fig. 1--Grid layout and a typical cell.

2.2.2.1 Continuity Equation, (10)

No contribution to Phase 1.

2.2.2.2 Equation of Motion, (11)

1. Axial Motion: In this case, referring to the
pressure integral in Eq. (11), $u_j = v$ and $n_j = -1, 0, +1$
are the axial components of the unit normal to the bottom,
sides and top of the cell respectively. Equation (11)
therefore gives for the Phase 1 contribution

$$\Delta_1 (mv) = P_b \pi \left(r_2^2 - r_1^2 \right) \Delta t - P_a \pi \left(r_2^2 - r_1^2 \right) \Delta t$$

where r_2 and r_1 are the radii of the outer and inner cell
surfaces, respectively, and P_b, P_a are the pressures on the
bottom and top cell faces. These pressures are obtained from
the initial (time n) values of cell pressures by a simple
average of the pressures in the adjacent cells:

$$P_b = \frac{P^n(k) + P^n(kb)}{2}$$

$$P_a = \frac{P^n(k) + P^n(ka)}{2} .$$

Making these substitutions gives the Phase 1 increment of
axial momentum

$$\Delta_1 (mv) = \left[\frac{P^n(kb) - P^n(ka)}{2} \right] \pi \left(r_2^2 - r_1^2 \right) \Delta t.$$

2. Radial Motion: To arrive at the radial equation
of motion, it is useful to consider a volume with the full
cell dimensions Δz and Δr in the z and r direction,
but with a small angular dimension $\Delta \theta$ instead of 2π (see

Fig. 2). Then one can compute the radial motion from Eq. (11) since the motion corresponds to a fixed direction in space. For use in Eq. (11), then, $u_j = u$, and $n_j = -1, 1, -\Delta\theta/2, -\Delta\theta/2,$ 0, 0 correspond to the radial components of the unit normal to the inner, and outer surfaces, the two side surfaces and the top and bottom, respectively. The pressure-integral contribution in Eq. (11) becomes

$$\frac{\Delta\theta}{2\pi} \Delta_1 (mu) = P_\ell r_1 \Delta\theta \Delta z \Delta t - P_r r_2 \Delta\theta \Delta z \Delta t + P_s \Delta r \Delta z \Delta\theta \Delta t$$

where P_ℓ, P_r and P_s are the left, right and side face pressures respectively and are defined in terms of time n cell pressures as follows.

$$P_\ell = \frac{P^n(k) + P^n(k\ell)}{2}$$

$$P_r = \frac{P^n(k) + P^n(kr)}{2}$$

and

$$P_s = \frac{P_\ell + P_r}{2}$$

$$= \frac{P^n(kr) + P^n(k\ell) + 2P^n(k)}{4}$$

In terms of these cell pressures, and using $\Delta r = r_2 - r_1$, the equation for radial motion becomes

$$\Delta_1 (mu) = \frac{P^n(k\ell) - P^n(kr)}{2} 2\pi\left(\frac{r_1 + r_2}{2}\right) \Delta z \Delta t .$$

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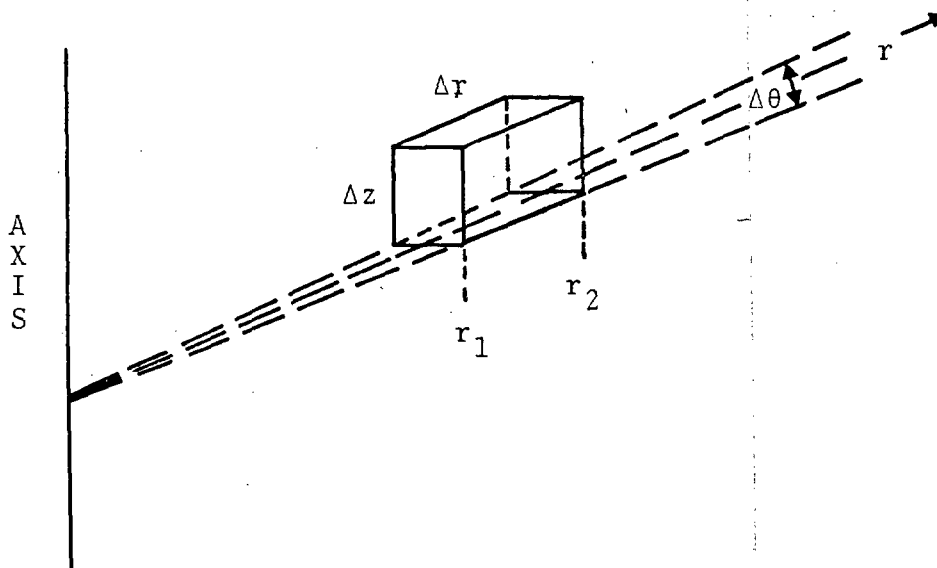
Fig. 2--Element of volume for discussion
of radial motion.

2.2.2.3 Energy Equation (12)

Computing the Phase 1 energy equation departs somewhat from the standard procedure which is used to compute the other integrals on the right in Eqs. (1) through (12). Specifically, rather than compute the gain in total cell energy, $m\left(E_m + \frac{1}{2}\left(u^2 + v^2\right)\right)$, by integrating the pressure forces on the cell surface, the original equation is reformulated to give a volume integral expression for the gain in internal energy, the kinetic energy increment having already been fixed by the preceding calculation of the velocity increments. The resulting equation is preferred because it is a simpler expression for E_m and because it has been used in most of our calculations and has led to satisfactory results. Starting with the Phase 1 part of Eq. (12),

$$\Delta_1\left(mE_T\right) = -\int_s Pu_i n_i ds \ .$$

and since the cell mass m is constant in Phase 1 and $E_T = E_m + \frac{1}{2} u_i u_i$ one can write

$$\Delta_1\left(mE_m\right) + u_i\Delta_1\left(mu_i\right) = -\int_s Pu_i n_i ds$$

$$= -\int_V \frac{\partial}{\partial x_i}\left(Pu_i\right)dV$$

$$= -\int_V P\frac{\partial u_i}{\partial x_i}dV - \int_V u_i\frac{\partial P}{\partial x_i}dV$$

$$= -P\int_V \frac{\partial u_i}{\partial x_i}dV - u_i\int_V \frac{\partial P}{\partial x_i}dV$$

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where P and u_i in the last line are average values over the cell volume. But from the Phase 1 momentum equation

$$\Delta_1 \left(mu_i\right) = - \int_S Pn_i ds$$

or

$$\Delta_1 \left(mu_i\right) = - \int_V \frac{\partial P}{\partial x_i} dV$$

so that the last terms on each side of the energy equation cancel and the resulting expression for the gain in internal energy is

$$\Delta_1 \left(mE_m\right) = - P \int_V \frac{\partial u_i}{\partial x_i} dV .$$

The difference form is

$$m\Delta E_m = - \Delta t \; P^n(k) \left(\frac{\partial u_i}{\partial x_i}\right)^{n+\frac{1}{2}} \pi \left(r_2^2 - r_1^2\right) \Delta z$$

where $\left(\partial u_i/\partial x_i\right)^{n+1/2}$ is obtained by averaging the time (n) (pre-Phase 1) and time $(n+1)$ values obtained from the Phase 1 momentum calculation, i.e.,

$$\left(\frac{\partial u_i}{\partial x_i}\right)^{n+\frac{1}{2}} = \frac{\left(\frac{\partial u_i}{\partial x_i}\right)^n + \left(\frac{\partial u_i}{\partial x_i}\right)^{n+1}}{2}$$

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and, for example, the time (n) value of this divergence is
determined from the calculation

$$\frac{\partial u_i}{\partial x_i} = \frac{1}{r} \frac{\partial}{\partial r} (ru) + \frac{\partial v}{\partial z}$$

$$\left(\frac{\partial u_i}{\partial x_i}\right)^n = \frac{1}{r_c} \left(\frac{r_r u^n(kr) - r_\ell u^n(k\ell)}{2\Delta r}\right) + \frac{v^n(ka) - v^n(kb)}{2\Delta z}$$

In which $r_c = \left(r_1 + r_2\right)/2$, $r_r = r_2 + \Delta r/2$, $r_\ell = r_1 - \Delta r/2$ are
the radii of cell centers for the central, right and left
cells respectively.

This completes the discussion of the Phase 1 calcula-
tion, for a typical cell, of the momentum and internal energy
increments due to the pressure forces on the cell. The two
momentum increments are used to determine the velocity incre-
ments by dividing by cell mass, and one has, at the end of
Phase 1, new values of $E_m(k)$, $u(k)$, $v(k)$.

2.2.2.4 Extension of Phase 1 to Mixed Cells

The changes required to extend Phase 1 to multi-material
cells are minor. The velocity increments are determined as
before, it being assumed that all the materials in a mixed
cell have a common velocity (u, v). The total increase in the
internal energy within the cell is computed as before, and
this energy is partitioned among the materials by assuming
that energy increments are proportional to fractional volumes
occupied by the various materials. These fractional volumes
are computed by dividing the mass of each material by its
density. Densities and masses are updated in the equation
of state and Phase 2 routines, respectively.

2.2.3 Phase 3—The Effects of Deviatoric Stresses

2.2.3.1 Continuity Equation, (10)

No contribution in Phase 3.

2.2.3.2 Equation of Motion, (11)

1. Axial Motion: The Phase 3 contribution from Eq. (11) is

$$\Delta_3 \left(mu_j \right) = \Delta t \int_s s_{ij} n_i \, ds$$

and for the axial motion

$$u_j = v \ .$$

Here $s_{ij} n_i$ is the axial component of the total stress on a surface. For a torus with rectangular section, $s_{ij} n_i$ on the various faces is

$$s_{zz}^t \quad \text{cell top}$$

$$- s_{zz}^b \quad \text{cell bottom}$$

$$s_{rz}^r \quad \text{outer cell surface}$$

$$- s_{rz}^\ell \quad \text{inner cell surface}$$

where zz and rz subscripts denote normal and shear stresses. The top, bottom, right and left surfaces are indicated by t, b, r, and ℓ respectively. The equation for axial

motion is therefore

$$\Delta_3 (mv) = \left(s_{zz}^t - s_{zz}^b \right) \pi \left(r_2^2 - r_1^2 \right) \Delta t + s_{rz}^r \, 2\pi r_2 \, \Delta z \, \Delta t$$

$$- s_{rz}^\ell \, 2\pi r_1 \Delta z \, \Delta t \; .$$

2. <u>Radial Motion</u>: For the radial motion, consider the element of mass depicted in Fig. 2. Then in the Phase 3 part of Eq. (11)

$$\Delta_3 \left(mu_j \right) = \Delta t \int_s s_{ij} n_i ds$$

and

$$u_j = u \; .$$

The radial components $s_{ij} n_i$ of the stresses on the various faces of the mass element are

$$s_{zr}^t \quad \text{top of mass element}$$

$$- s_{zr}^b \quad \text{bottom}$$

$$s_{rr}^r \quad \text{right}$$

$$- s_{rr}^\ell \quad \text{left}$$

$$- s_{\theta\theta} (\tfrac{1}{2}\Delta\theta) \quad \text{front}$$

$$- s_{\theta\theta} (\tfrac{1}{2}\Delta\theta) \quad \text{back} \; .$$

The mass of the element is $m\Delta\theta/2\pi$ where m is the cell mass. Then

$$\frac{\Delta\theta}{2\pi}\,\Delta_3(mu) = \left(s_{zr}^t - s_{zr}^b\right)\left(r_2^2 - r_1^2\right)\frac{\Delta\theta}{2}\,\Delta t + s_{rr}^r\,r_2\,\Delta\theta\,\Delta z\,\Delta t$$

$$- s_{rr}^\ell\,r_1\,\Delta\theta\,\Delta z\,\Delta t - s_{\theta\theta}\Delta z\,\Delta r\,\Delta\theta\,\Delta t$$

or (multiplying by $2\pi/\Delta\theta$) the equation for radial motion is

$$\Delta_3(mu) = 2\pi\,\Delta t\left[\frac{1}{2}\left(s_{zr}^t - s_{zr}^b\right)\left(r_2^2 - r_1^2\right)\right.$$

$$\left. + s_{rr}^r\,r_2\Delta z - s_{rr}^\ell r_1\Delta z - s_{\theta\theta}\Delta z\,\Delta r\right]\ .$$

2.2.3.3   Energy Equation, (12)

In the Phase 3 term from Eq. (12),

$$\Delta_3\left(mE_T\right) = \Delta t\int_S s_{ij}u_j n_i ds\ ,$$

$s_{ij}u_j n_i$ is the work rate per unit surface area, which for the various faces of the torus, is

$$\left(s_{zz}^t v^t + s_{zr}^t u^t\right)\quad \text{cell top}$$

$$-\left(s_{zz}^b v^b + s_{zr}^b u^b\right)\quad \text{cell bottom}$$

$$\left(s_{rz}^r v^r + s_{rr}^r u^r\right)\quad \text{outer cell surface}$$

$$-\left(s_{rz}^\ell v^\ell + s_{rr}^\ell u^\ell\right)\quad \text{inner cell surface}$$

where t, b, r, ℓ denote stresses and velocities at the top, bottom, right and left cell faces respectively. The interface

velocities are determined from a simple average of the time
n velocities in the two cells, e.g.,

$$v^t = \frac{v^n(k) + v^n(ka)}{2}$$

$$u^\ell = \frac{u^n(k) + u^n(k\ell)}{2} .$$

The equation for the Phase 3 change in total cell energy is

$$\Delta_3\left(mE_T\right) = \left(s_{zz}^t v^t + s_{zr}^t u^t - s_{zz}^b v^b - s_{zr}^b u^b\right)\pi\left(r_2^2 - r_1^2\right)\Delta t$$

$$+ \left(s_{rz}^r v^r + s_{rr}^r u^r\right) 2\pi\, r_2 \Delta z\, \Delta t$$

$$- \left(s_{rz}^\ell v^\ell + s_{rr}^\ell u^\ell\right) 2\pi\, r_1 \Delta z\, \Delta t .$$

This completes the discussion of the Phase 3 calculation,
for a typical cell, of the momentum and total energy increments
due to the deviator stresses acting on the cell. The two momen-
tum increments are used to determine the velocity increments
by dividing by cell mass. This fixes the Phase 3 change in
cell kinetic energy $\frac{1}{2} u_i u_i$ so that the internal energy E_m
can be calculated using the updated value of total energy
$E_T = E_m + \frac{1}{2} u_i u_i$. At the end of Phase 3 there are new values
of $E_m(k)$, $u(k)$, $v(k)$ as updated by Phase 3. As in Phase 1,
the change in internal energy for mixed cells is partitioned
among the materials in proportion to their partial volumes.

2.2.4 Phase 2—The Effects of Transport

The purpose of this section is to describe how the
transport of mass, momentum and energy from cell to cell is
accounted for in the code. This is done by calculating the

integrals in the last terms of Eqs. (10) through (12). In the discussion below, primary attention is given first to the case of transport between pure cells. At the conclusion of this discussion, the special provisions required to treat mixed cells are described.

2.2.4.1 Continuity Equation, (10)

The transport mass is

$$\Delta_2(m) = - \Delta t \int_S \rho u_i n_i ds$$

and is determined for each cell face from

$$\delta m = - \rho^d \, \bar{u}_i A_i \Delta t \ .$$

ρ^d is the density of the cell from which the mass moves (donor cell), A_i is the area of the face and $\bar{u}_i$ is an interpolated value of the velocity component normal to A_i representing approximately the velocity at the interface at the end of the time step. For example, considering cells k and ka above,

$$\bar{v} = \frac{\frac{1}{2}\left[v^n(k) + v^n(ka)\right]}{\left[1 + \frac{v^n(ka) - v^n(k)}{\Delta z} \Delta t\right]} \ .$$

Calculated transport masses are subtracted from the donor cell mass and added to the acceptor cell mass. This updating is done after the transport terms have been calculated; all transport terms are computed using the pre-transport quantities.

2.2.4.2 Equation of Motion, (11)

1. Axial Motion: The term in Eq. (11) for axial momentum transport is

$$\Delta_2 \left(m u_j \right) = - \Delta t \int_S \left(\rho u_j u_i \right) n_i ds \ .$$

At each face of the cell the transport specific momentum, u_j, is taken to be the axial velocity of the cell from which the mass moves (donor cell, index kd), i.e.,

$$u_j = v^n(kd) \ .$$

Since the various faces of the cell have different donor cells it is convenient to express the momentum transport for each face

$$\delta(mv) = v^n(kd) \delta m$$

where

$$\delta m = - \rho^d \, \bar{u}_i A_i \Delta t$$

is the mass which is transported across the cell face, as given in the preceding mass transport calculation. Note that $\delta(mv)$ is the axial momentum and that δm is the mass transport in either the r or z direction, depending on which face of the cell is being computed.

2. Radial Motion: Again from Eq. (11),

$$\Delta_2 \left(m u_j \right) = - \Delta t \int_S \left(\rho u_i u_j \right) n_i ds$$

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and, by analogy with the axial case, the equation for the transport of radial motion across an interface is

$$\delta(mu) = u^n(kd)\,\delta m$$

where $u^n(kd)$ is the time n velocity of the donor cell and $\delta m = -\rho^d\,\bar{u}_i A_i \Delta t$ is the mass which is transported across the cell face in question, as computed in the continuity equation (above).

2.2.4.3 Energy Equation (12)

The Eq. (12) expression for the transport of energy is

$$\Delta_3\left(mE_T\right) = -\Delta t \int_S \left(\rho u_i E_T\right) n_i\, ds \ .$$

To evaluate this integral, the transported specific energy is taken to be that of the donor cell, kd, i.e.,

$$E_D = E_m^n(kd) + \frac{1}{2}\left[\left(u^n(kd)\right)^2 + \left(v^n(kd)\right)^2\right]$$

and the total energy which is transported across a given interface is therefore the product of this specific energy and the associated transport mass which was computed above in the continuity equation, i.e.,

$$\Delta_3\left(mE_T\right) = E_D\,\delta m \ .$$

Once the mass, momentum and energy transports are known for all faces of the cell, the cell quantities can be updated for these effects. New cell velocities are determined by dividing the new momenta by updated cell mass. This fixes

the new kinetic energy so that the internal energy can be calculated from the known total energy.

2.2.4.4 Extension of Phase 2 to Mixed Cells

The procedure used to treat mixed cells is an extension of the preceding method for transport between pure cells. Specifically, the mass, say for material b, which is transported across a given interface is, in analogy with the result in Section 2.2.4.1 above, just

$$\Delta_2\left(m_b\right) = \Delta t \int_s \rho_b \, u_i \, n_i \, ds$$

where now ρ_b is the donor cell density for material b and the surface integral is now over just that part of the surface area belonging to material b. The determination of the density ρ_b is made in the pressure iteration, to be the subject of Section V; the determination of the fractional surface area belonging to a given material is part of the discussion in Section IV of the method for computing the intersection of material interfaces and cell boundaries.

The momentum transport, in analogy with the pure cell case, is just this mass transport multiplied by the appropriate velocity component (axial or radial) of the donor cell material.

The energy transport is the mass transport multiplied by the energy per unit mass, kinetic plus internal. The kinetic energy per gram is the same for all materials in the donor cell, but the internal energies are generally different (internal energies were updated in Phase 1 and Phase 3 as described in prior sections).

As in the case of pure cells, final values of cell variables are easily determined once all the transport quantities are known for the cell. The new cell velocity components are determined by dividing the new momenta by the updated cell mass. This fixes the new kinetic energy for each material; the internal energy for each material is then chosen to conserve total energy, kinetic plus internal, for the individual materials.

2.3 THE EQUATION OF STATE

2.3.1 The Determination of P

The equation of state used in HELP, for initially condensed materials, is that due to J. H. Tillotson,[5] modified to give a smooth transition between condensed and expanded states. For the condensed states, i.e., when $\rho/\rho_0 > 1$, or for any cold states, $I < I_s$, the equation has the form

$$P = P_C = \left[a + \frac{b}{\dfrac{I}{I_0 \eta^2} + 1} \right] I\rho + A\mu + B\mu^2$$

For expanded hot states, i.e., when $\rho/\rho_0 < 1$ and $I > I_s'$ the equation of state has the form

$$P = P_E = aI\rho + \left[\frac{bI\rho}{\dfrac{I}{I_0 \eta^2} + 1} + A\mu e^{-\beta\left(V/V_0 - 1\right)} \right] e^{-\alpha\left(V/V_0 - 1\right)^2}$$

A smooth transition between the condensed and expanded states is insured by a transition equation for the intermediate region defined by $I_s < I < I_s'$ and $\rho/\rho_0 < 1$. This blended portion

of the equation of state has the form

$$P = \frac{\left(I - I_s\right) P_E + \left(I_s' - I\right) P_C}{I_s' - I_s}$$

In these equations, p, I and ρ are pressure, specific internal energy and mass density respectively, $\eta = \rho/\rho_0 = \mu + 1 = V_0/V$, and ρ_0, a, b, I_0, I_s, I_s', A, B, ρ and ρ_0 are constants for the particular material. The values of these constants for nineteen materials are given in Table I.

For mixed cells, additional considerations are require to determine the cell pressure and the densities of the various materials within the cell. This problem is discussed as Section V.

2.3.2 The Determination of Deviatoric Stresses

The deviatoric stress increments, ds_{ij}, are determined by using the elastic relation

$$ds_{ij} = 2G d\varepsilon_{ij}'$$

where G is the modulus of rigidity and $d\varepsilon_{ij}'$ are the increments of deviatoric strain. When such an increment of stress causes the yield criterion to be violated each stress component is proportionately reduced to bring the stress state normally back to the yield surface. A variable yield strength

$$Y = \left(K_0 + K_1 \mu + K_2 \mu^2 \right)\left(1 - E/E_M \right)$$

is defined, to account for the increase in strength at high pressures and the decrease in strength at elevated temperatures.

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TABLE I
HELP EQUATION OF STATE CONSTANTS[1]

Material Code Number	Material	a	b	A $dynes/cm^2$	B $dynes/cm^2$	E_0 ergs/g	α	β	E_S	ρ g/cm^2	E_S'
1	W	.5	1.04	3.08×10^{12}	2.5×10^{12}	$.225 \times 10^{12}$	10	10	1.11×10^{10}	19.17	5.6×10^{10}
2	Cu	.5	1.5	1.39	1.1	.325	5	5	1.38	8.9	6.9
3	Fe	.5	1.5	1.28	1.05	.095	5	5	2.44	7.9	10.2
4	Al	.5	1.63	.75	.65	.05	5	5	3.0	2.7	15.0
5	Be	.55	.62	1.17	.55	.175	5	5	10.0	1.8	46.0
6	Ti	.5	.60	1.03	.5	.07	5	5	3.5	4.5	12.5
7	Ni	.5	1.33	1.91	1.5	.09	5	5	2.85	8.9	9.4
8	Mo	.5	1.02	2.71	1.65	.045	5	5	2.8	10.2	9.0
9	Th	.4	.86	.53	.5	.025	9	.88	2.0	11.7	8.0
10	Pb	.4	2.4	.466	.15	.02	10	2	.26	11.3	$.97 \times 10^{12}$
11	CH_2	.6	2.0	.075	.02	.07	10	5	2.4	.9	18.0
12	Granite[2]	.5	1.3	.60	.00	.16	5	5	3.5	2.7	.18
13	Andesite[2]	.5	1.3	.34	.28	.16	5	5	3.5	2.7	.18
14	Wet Tuff	.5	1.3	.10	.06	.11	5	5	3.2	2.0	.16
15	Dry Tuff	.5	1.3	.045	.03	.06	5	5	3.2	1.7	.18
16	Oil Shale	.5	1.0	2.8	.11	.11	5	5	3.2	2.3	.16
17	Dolomite	.5	.6	.85	.30	.10	5	5	2.5	2.8	.14
18	Limestone	.5	.6	.4	.67	.10	5	5	2.5	2.7	.14
19	Halite	.5	.6	.25	.30	.05	5	5	2.0	2.2	.15

[1]Dienes, J. K., et al., "An Eulerian Method for Calculating Strength Dependent Deformation," General Atomic Report GAMD-8497, February 1968.

[2]For these two materials the constants are valid only for pressures less than 150 kbar. For a formulation of the high pressure equation of state of these materials, see Allen, R. T., "Equation of State of Rocks and Minerals," General Atomic Report GAMD-7834, March 1967.

2.3.3 Material Failure in Tension

The failure criterion is a simple one based upon relative volume. When the relative volume in a cell reaches a certain value which is greater than a specified maximum distension, that cell is said to fail and any computed tensions are zeroed out.

2.4 OTHER FEATURES OF THE NUMERICAL METHOD

2.4.1 Grid Boundaries and Axis

Additional specification is required for cells which border the grid boundaries or axis of symmetry, because necessary quantities are not defined for neighbor cells which would be outside the grid. The code provides for a transmittive boundary at the top and right, an option of a transmittive or reflective boundary at the bottom and a reflective boundary at the axis. Boundary conditions for border cells are then derived by assuming (fictional) neighbor cells outside the grid. For transmittive boundaries the flow variable are the same in the fictional cell as in the border cell, and for reflective boundaries the state is assumed to be the same except that the velocity component normal to the boundary has the opposite sign.

2.4.2 Rezone

A rezone subroutine can be triggered by mass motion out of the right or top grid boundaries. Four cells are combined into one, starting at the lower left corner ($z = 0$, $r = 0$) of the grid and new cells are added at the top and right. The total number of cells and their shape is held constant. In combining cells during rezone, total mass, momentum and energy

are conserved. The net effect is a slight increase in internal energy, corresponding to the kinetic energy which the four cells had in a coordinate system moving with their center of mass.

In some cases, and specifically when a shock passes through a transmittive grid boundary, it has been found necessary to rezone (thus keeping the entire disturbance within the new grid) to avoid undesirable oscillations in the calculation. The oscillations apparently indicate that a simple transmittive boundary condition does not adequately simulate the effect of additional material.

2.4.3 Variable Zoning

Although the discussion proceeded with no special reference to variable zoning, the code has the capability of computing problems in which the zone size is variable. If constant zones are not specified, the cell sizes must be supplied as a part of the SETUP deck. It is possible to call for variable cell size in one direction and constant in the other.

The zones should be selected so that the discontinuities in size between adjacent cells are not excessive and the aspect ratio of the cells are not too extreme. It was found that an initially spherical explosion loses its symmetry in the course of a calculation when the ratio of the cell edges is greater than four to one, and that satisfactory symmetry is obtained when the ratio is two to one. Cell size variations are usually held to less than 10 percent between adjacent cells.

III. NUMERICAL METHOD FOR ELASTIC-PLASTIC EFFECTS

3.1 INTRODUCTORY REMARKS

The effects of material strength are computed as
Phase 3, and when this phase is simply by-passed the remaining
code solves problems in compressible fluid dynamics. Phase 3
adds the resistance to shear which characterizes solids.

The Phase 3 determination of strength effects is it-
self divided into two principal parts. One of these, the
effects of (predetermined) deviatoric stresses in changing
the material velocity field and the material specific inter-
nal energies, has been described above as Section 2.2.3.
The second part, discussed below, is the determination of
these stress deviators, given the velocity field and the
material constitutive equations.

3.2 DETERMINATION OF STRAIN RATE DEVIATORS

As a first step in computing stress deviators, the
instantaneous strain-rate deviators are computed from the
velocity field, i.e., for cylindrical coordinates

$$\dot{\varepsilon}_{zz} = v_y - \frac{1}{3}\left(u_x + v_y + \frac{u}{x}\right)$$

$$\dot{\varepsilon}_{rr} = u_x - \frac{1}{3}\left(u_x + v_y + \frac{u}{x}\right)$$

$$\dot{\varepsilon}_{rz} = \left(u_y + v_x\right)/2$$

Here the necessary space derivatives of the velocity field
are computed (centered at cell centers) in a straightforward

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manner, i.e.,

$$v_y = \frac{v(ka) - v(kb)}{2\Delta y}$$

$$v_x = \frac{v(kr) - v(kl)}{2\Delta x}$$

etc.
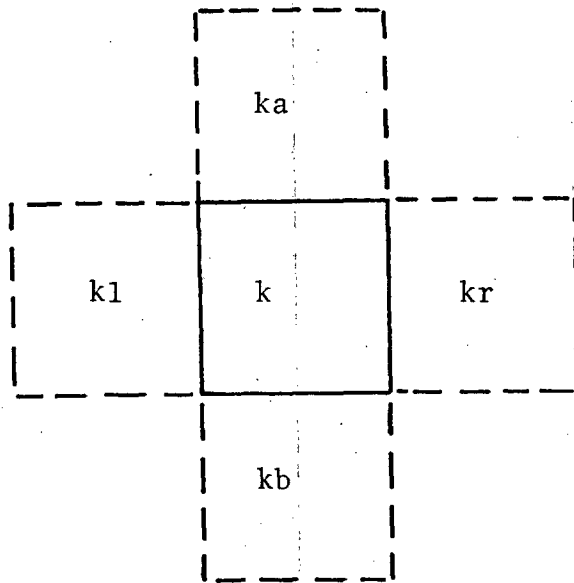
But the above calculation gives only the strain rate devia-
tor for that material which is at cell center at the beginning
of the time step, and it is necessary to recognize that con-
stitutive equations are to be applied only along a particle
path. To this end, one must take a convective derivative.
Consider the time N position P^N of a particle which, at
time N+1, will be at cell center. This point will be offset
from cell center by

$$\delta x = -u(k) \, \Delta t$$

$$\delta y = -v(k) \, \Delta t \, .$$
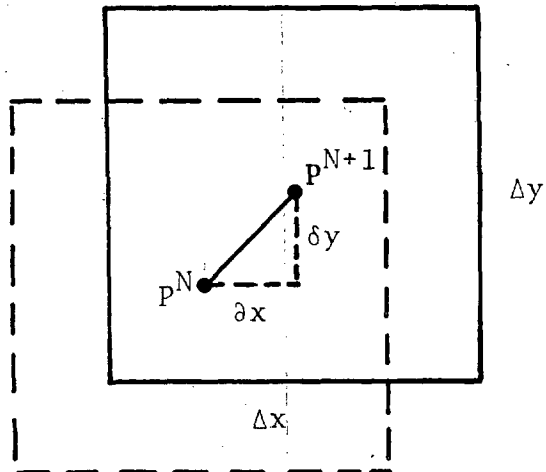
Now we wish to determine, by interpolation using the value of
the strain rate deviators at cell centers, the strain rate

deviators at the offset point P^N. This is done by weighting the contribution of neighboring cells in proportion to their overlap areas with a rectangle of cell dimensions, centered at P^N. For example, in the above sketch, the weighting factors are

for cell k

$$WK = (\Delta x - |\delta x|)(\Delta y - |\delta y|)$$

for cell in horizontal direction

$$WKH = |\delta x|(\Delta y - |\delta y|)$$

for cell in vertical direction

$$WKV = |\delta y|(\Delta x - |\delta x|)$$

for cell in diagonal direction

$$WKD = |\delta x||\delta y|$$

so that the resulting interpolated value of a strain rate deviator component, say $\dot{\varepsilon}_{rz}$, is given by

$$\dot{\varepsilon}_{rz} = \frac{(WK)\dot{\varepsilon}_{rz}(k) + (WKH)\dot{\varepsilon}_{rz}(kh) + (WKV)\dot{\varepsilon}_{rz}(kv) + (WKD)\dot{\varepsilon}_{rz}(kd)}{WK + WKH + WKV + WKD}$$

Having thus determined $\dot{\varepsilon}_{zz}$, $\dot{\varepsilon}_{rr}$, $\dot{\varepsilon}_{rz}$ for the mass which will be at the center of cell k at the end of the time step, we can now update the strain deviators. Say, for ε_{rz}

$$\Delta\varepsilon_{rz} = \dot{\varepsilon}_{rz} \Delta t$$

3.3 DETERMINATION OF STRESS DEVIATORS

These strain increments are next used in the material constitutive equation, to update the stress deviator tensor s_{ij}. To do this one needs the time N stress deviator for the particle in question and these are determined (from stored cell centered values of the s_{ij} at time N) by an identical area weighting procedure as described above for strain rate deviators.

For an elastic-plastic material the following calculation then (provisionally) updates the s_{ij} according to

$$s_{ij}^{N+1} = s_{ij}^{N} + 2G \, \Delta\varepsilon_{ij}$$

where G is the material rigidity modulus, i.e., the three components of interest are given by

$$s_{zz}^{N+1} = s_{zz}^{N} + 2G \, \Delta\varepsilon_{zz}$$

$$s_{rr}^{N+1} = s_{rr}^{N} + 2G \, \Delta\varepsilon_{rr}$$

$$s_{rz}^{N+1} = s_{rz}^{N} + 2G \, \Delta\varepsilon_{rz}$$

The provision (above) is that the yield condition for the material not be violated, i.e., one tests to see if the second invariant exceeds the yield condition

$$s_{ij}s_{ij} = s_{zz}^{2} + s_{rr}^{2} + s_{\theta\theta}^{2} + 2s_{rz}^{2} > 2Y^{2}$$

Here $s_{\theta\theta} = -\left(s_{rr} + s_{zz}\right)$ and Y is the yield strength in shear. As pointed out in Section 2.3, the yield point can be some function of the local thermodynamic state (i.e., of

ρ and E). If the yield point is exceeded during a time step, as would occur in plastic deformation, all of the stresses are proportionately reduced to cause the yield condition to be satisfied. The second invariant being smaller than $2Y^2$, on the other hand, corresponds to elastic deformation, for which the stresses are not reduced.

The preceding calculation updates the s_{ij}, giving values of these quantities at cell centers (as corrected for material convection.) For the use of stress deviators in updating velocities and energies, however, the method described as Section 2.2.3 assumes values of these quantities at cell surfaces rather than cell centers. These cell-surface quantities are obtained by merely averaging the corresponding quantities at the centers of the two cells in question.

3.4 MODIFICATIONS FOR GRID BOUNDARIES AND AXIS

The description given above applies to normal cells within the grid. Additional statements must be made to cover grid boundaries and the axis of symmetry.

Grid boundaries may be reflective (bottom grid boundary) or transmittive (bottom, right or top). In calculating the strain rate deviators for reflective cells (from the velocity field) it is assumed that the cell which would be outside the grid, has the same tangential component of velocity and an equal and opposite normal component of velocity, as the cell within the grid. Thus, for cell k at a reflective bottom grid boundary, the assumed cell just below k has velocities

$$u = u(k)$$

$$v = -v(k)$$

These velocities are then used in an otherwise standard (above)

calculation of the strain rate deviator tensor for cell k. For transmittive boundary cells, one assumes that the cell outside the grid has both of its velocity components equal to the velocity of the cell just inside the boundary.

$$u = u(k)$$

$$v = v(k)$$

With these assumptions, it is then possible to compute strain rate deviators for the border cells. For either transmittive or reflective grid boundaries, the normal stress at the grid boundary is taken to be that of the border cell. The tangential stress is zero for reflective boundaries and is equal to that in the border cell for transmittive boundaries; e.g., for a reflective boundary at grid bottom

$$SNB = s_{zz}(k) \qquad \text{stress normal bottom}$$

$$STB = 0 \qquad \text{stress tangent bottom}$$

and for a transmittive boundary at the right of the grid

$$SNR = s_{rr}(k) \qquad \text{stress normal right}$$

$$STR = s_{rz}(k) \qquad \text{stress tangent right}$$

The axis of symmetry is essentially a reflective grid boundary. In calculating strain rate deviators for the column of cells next to the axis, the cell "to the left" is assumed to have the same axial velocity and an equal and opposite radial velocity, i.e.,

$$v = v(k)$$

$$u = -u(k)$$

Stresses for the axial boundary of this column of cells are simply set to zero in the program, since the cell area over which these stresses act is zero and it is the product of stresses and areas which is of importance in computing the effects on cell velocities and energy (Section 2.2.3).

3.5 MODIFICATIONS FOR MATERIAL FREE SURFACES

Cells within the grid which contain a material free surface require special consideration. In general, such cells are only partly filled with material and are partly void. The program complications that would be involved by detailed treatment of such partially filled cells are avoided by simply insuring that such cells have the same velocity and specific internal energy as their highest pressure neighbor cell. This is done in the GLUE routine (see Section 7.1.10), after Phase 1 is executed. The glueing of two such cells is done by conserving total momentum and energy of the pair of cells, and leaving the mass within each cell the same. After glueing, the two cells have the same velocity and specific internal energy. In Phase 3, then, the stress deviators are assumed to be zero within a partially filled free surface cell and the stresses at the boundaries between this cell and its neighbors are also assumed to be zero. The combined effect of glueing and neglecting actual stresses would be sufficient to cause the free surface cell material to have the same velocity as the main body of material.

IV. THE PROPAGATION OF TRACER PARTICLES
AND THEIR USE TO DESCRIBE MATERIAL INTERFACES

4.1 INTRODUCTORY REMARKS

The most complicated logic of the HELP code is found in the subroutines[1] dealing with the definition and use of material interfaces. Conceptually the task is not difficult. The idea of moving tracer particles with a locally defined velocity and of using them to describe the interface between materials is a simple one. Likewise, employing the partial cell boundary areas determined by interface position to compute the mass flux of materials is a very natural use of these interfaces. The complexity of the task comes with making the treatment completely general. As the code is written, there are no restrictions on the direction of the flow, on the shape of the material interface, on the number of times an interface can cross a cell boundary, or on the number of materials contained in a cell.

It is hoped that the present section will aid the user in understanding the organization and logic of these routines. In particular, this section covers the definition of the material interfaces, the computation of fractional cell boundary areas for each material in a mixed cell, and the use of these fractional areas in defining the mass transport terms. The creation of mixed and pure cells, and the propagation of the tracer particles that define the package[2] boundaries will also be discussed.

---

[1] INFACE, NEWMIX, NEWRHO, FLUX.

[2] To distinguish the INFACE and PH2 (Phase 2) functions, it should be noted that the mass transports (for mixed cells) are only computed and stored in INFACE. The actual transport of all quantities, and the associated updating of cell variables, is performed in PH2 for both mixed and pure cells.

4.2 DEFINING MATERIAL INTERFACES

A set of massless tracer particles are initially posi-
tioned along the boundary of each material package. These
tracers are numbered such that the package is on the left as
one proceeds between any two consecutive tracers. The material
interface therefore is defined by the line segments connecting
these tracers. The initial density of tracers is controlled
by input variables and can be increased in a specified region
as the calculation progresses. (See description of ADDTCR,
Section 7.1.1.)

To prevent the boundaries of adjacent packages from
crossing, the tracers along the common boundary coincide
exactly. Because these identical points are moved with
exactly the same velocity, they remain superimposed during
the entire calculation.

To allow for spatially disconnected subpackages, the
code employs the following convention. The last point of a
package or subpackage is a dummy point and is identical to
the first point of the package or subpackage. The line
drawn between the last two points is therefore not considered
part of the package boundary. Likewise that last point is
assumed to be disconnected from the first point of the next
subpackage if such a subpackage exists.

4.3 COMPUTING FRACTIONAL AREAS

4.3.1 Problem Definition

When an Eulerian cell is cut by the material N interface, we are interested in knowing what fraction of each side of the cell should be associated with material N.
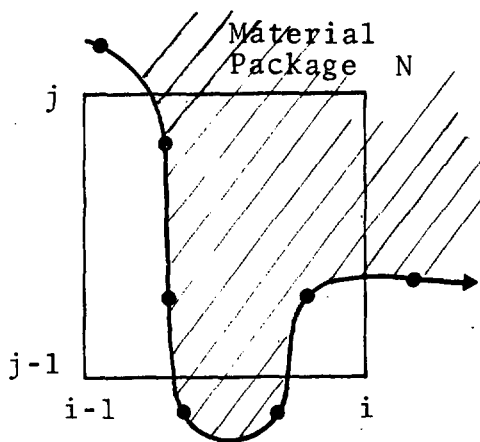
For example, the fractional areas associated with material N in the diagram above are as follows:

$$\text{Left} = 0.$$

$$\text{Top} \sim .8 \; \pi\left(x_i^2 - x_{i-1}^2\right)$$

$$\text{Right} \sim .7\left(y_j - y_{j-1}\right) 2\pi \; x_i$$

$$\text{Bottom} \sim .3\pi\left(x_i^2 - x_{i-1}^2\right)$$

These fractional areas are used to compute the mass flux of material N across each side of the mixed cell. (Actually, for each mixed cell it is necessary to compute and store only

the fractional areas for the right and top sides since the cells below and on the left provide the information for the other two sides.) The method given below for computing these fraction-areas puts no restrictions on how many inter-faces are in a cell or on how many times a single interface crosses a cell or cell boundary. It does, however, assume that a material package boundary never crosses itself, and experience has shown that when it does the logic of INFACE breaks down. This is not a limitation of the method since the boundary will not cross itself unless the tracers become very sparse, and this can be prevented by adding new tracers, as discussed in the description of the subroutine ADDTCR.

4.3.2   Computing Intersections of Interfaces with Grid Lines

Subroutine INFACE considers consecutively (two at a time) the tracer points which circumscribe each material package in a counter-clockwise direction. For simplicity, the coordinates of the tracers are in grid line units rather than centimeter units. Thus the coordinates of a pair of tracers immediately indicate whether they straddle one or more cell boundaries.

TX and TY are the FORTRAN variable names of a tracer's x and y coordinates, respectively. The arrays are doubly dimensioned; the first dimension identifies the material package, the second gives the sequential ordering of the points. In the example shown below, the indices indicate that both tracer points belong to material package one (first index) and that they are consecutive points (second index), namely the 20th and 21st points describing package one. The x-coordinates indicate the line through the points does not inter-sect a vertical grid line, whereas the y-coordinates indicate the line through the points intersects the fourth horizontal grid line.

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$$TX(1,20) = 2.5$$
$$TY(1,20) = 3.5$$
$$TX(1,21) = 2.8$$
$$TY(1,21) = 4.6$$
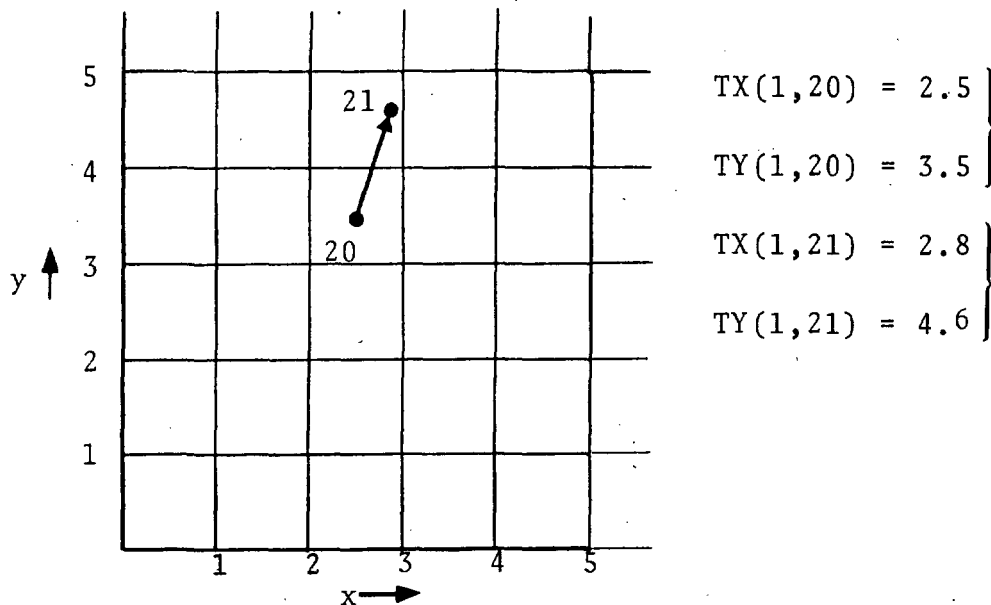
4.3.3 Defining Fractional Areas of Intersected Cell Boundaries

To continue with the above example, INFACE defines the point of intersection by a straight line interpolation between tracers, and uses it to define the fractional area of the top cell boundary associated with material package one. The code recognizes from the sequential numbering of the points that material one is on the left of the point of intersection.

The FORTRAN variables FRACTP and FRACRT store the fraction areas of the top and right boundaries, respectively for all the materials of a mixed cell. These arrays are also doubly dimensioned. Given FRACTP(N,M), FRACRT(N,M), the first dimension, N, identifies the material package number, and the second dimension, M, is an index that links the mixed cell K to the special mixed cell arrays by the relation, M=FLAG(K)-100.

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If material package two is adjacent to material package one, the material two tracers are identical to the material one tracers but are in an opposite order. For example, package two tracers (say 30 and 31) that coincide with the package one tracers discussed previously would be as follows:

$$TX(2,30) = 2.8$$

$$TY(2,30) = 4.6$$

$$TY(2,31) = 2.5$$

$$TY(2,31) = 3.5$$

The fractional area to the right of the intersection point is associated with material package two, and is stored in FRACTP(2,M).

4.3.4 Defining Fractional Areas of Boundaries Not Intersected

Because the fractional area is a term in the flux equations applied to mixed cells, it must be defined for the right and top boundary of every mixed cell even if one or both are not intersected by a material interface. In that case the fractional area either equals the total cell boundary area or is zero, e.g.

Material Package 1

$$FRACTP(1,M) = \pi * \left(x_i^2 - x_{i-1}^2 \right)$$

$$FRACRT(1,M) = \left(y_j - y_{j-1} \right) 2\pi x_i$$

Material Package 2

$$FRACTP(2,M) = 0.$$

$$FRACRT(2,M) = 0.$$

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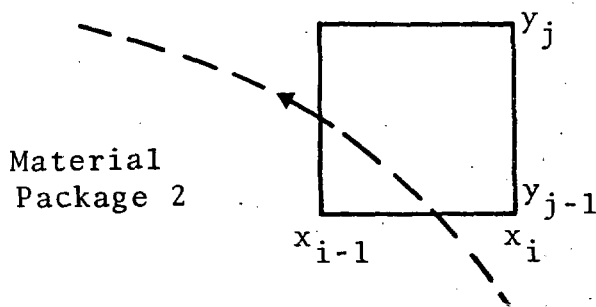
In the above example, none of the material of package two is transported across the right or top boundaries of the cell. On the other hand, the full cell area is used to compute the mass flux of the material of package one across these boundaries. There is clearly a need to systematically define the fractional areas of mixed cell boundaries that are not intersected by a material interface. The following discussion illustrates how the fractional area of these non-intersected boundaries are correctly defined by "presetting" them when the interface enters a cell and "resetting" them when it leaves.

The direction of an interface, given by the sequential order of the tracer points, determines whether one is entering a cell or leaving it. When entering a cell, and if crossing this cell boundary for the first time, the program processes the other sides of the cell in a clockwise order and presents their fractional areas to equal the total cell boundary area until it comes to a side that has already been crossed by the same material interface (i.e., a side which has an associated fractional area that is non-zero and is less than the total area.)

Case (1)
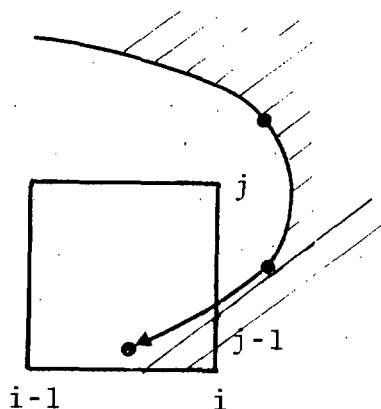
Fractional areas preset:

$$\text{Bottom} = \pi\left(x_i^2 - x_{i-1}^2\right)$$

$$\text{Left} = \left(y_j - y_{j-1}\right)2\pi\, x_{i-1}$$

$$\text{Top} = \pi\left(x_i^2 - x_{i-1}^2\right)$$

Case (2)
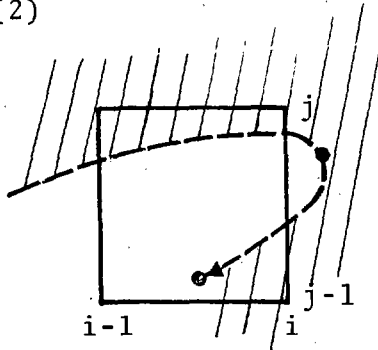
Fractional areas preset:

$$\text{Bottom} = \pi\left(x_i^2 - x_{i-1}^2\right)$$

Case (3)
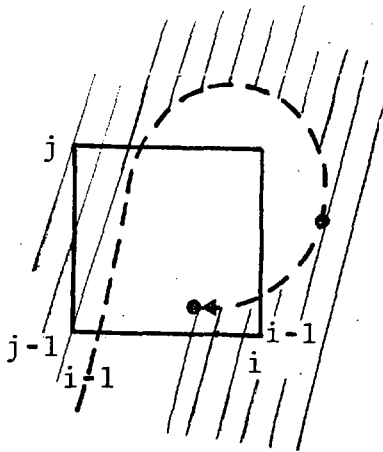
Fractional areas preset:

None

If the interface of material N crosses a cell boundary for the second time, the code does not preset of fractional areas when material N is between the two points of intersection (Case 4). If material N is outside the two points, the fractional areas are preset as described above (Case 5). The program senses that material N is between the points when the sum of the fractional area computed on the previous crossing(s) and the one computed currently is greater than the total area.

Case (4)
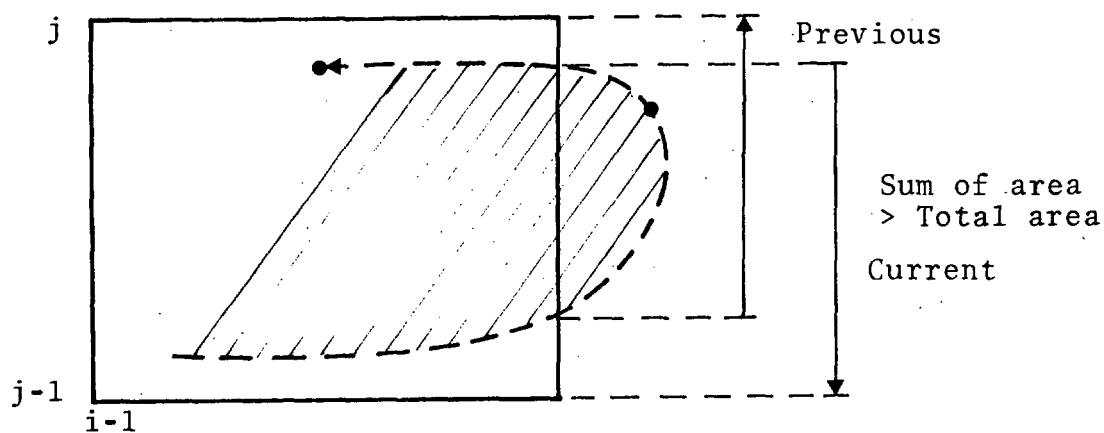
Fractional areas preset:

None

Case (5)
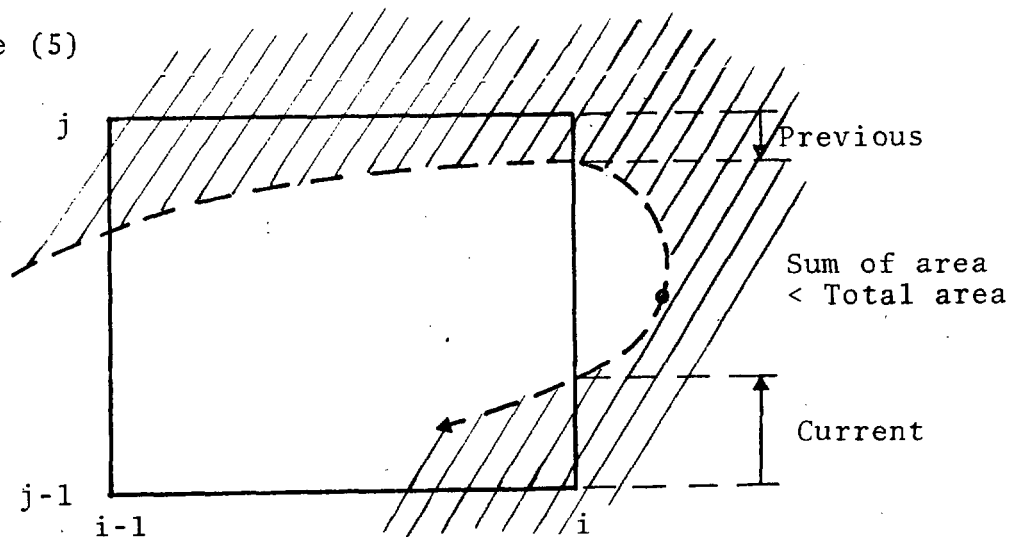
Fractional areas preset:

$$\text{Bottom} = \pi\left(x_i^2 - x_{i-1}^2\right)$$

Actually this rule holds regardless of how many times the interface crosses the boundary, as illustrated by Case 6 and Case 7.

Case (6)
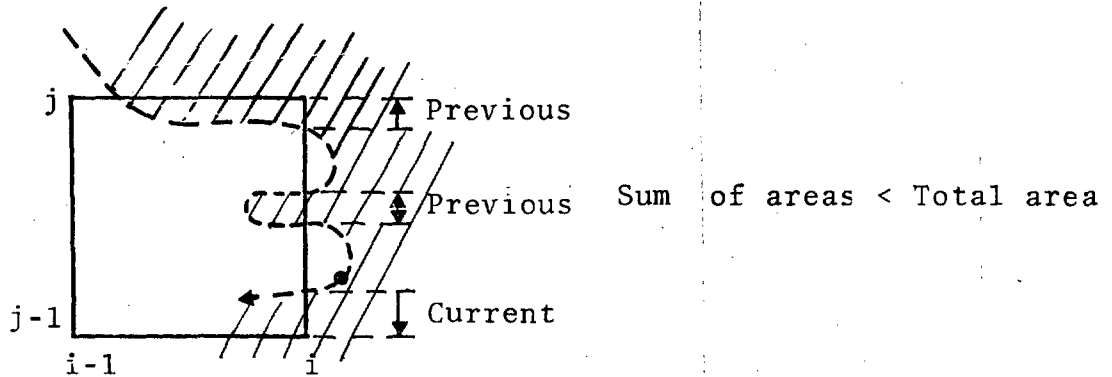
Previous

Previous   Sum of areas < Total area

Current

Fractional areas preset:

$$Bottom = \pi\left(x_i^2 - x_{i-1}^2\right)$$

$$Left = \left(y_j - y_{j-1}\right)2\pi\, x_{i-1}$$

Case (7)

Previous

Current   Sum of areas > Total area

Previous
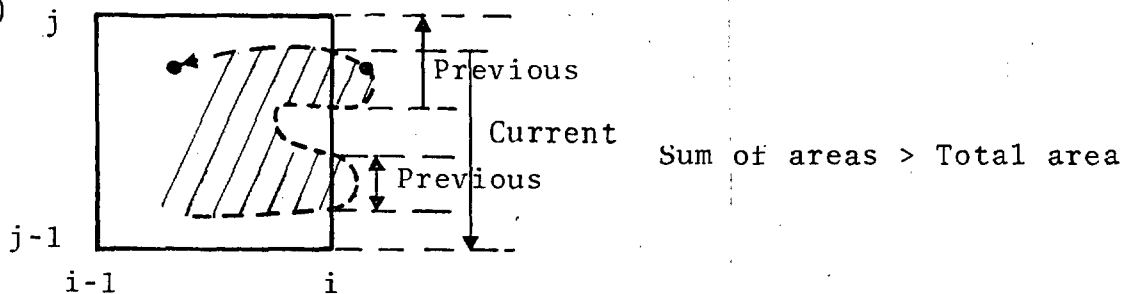
Fractional areas preset:

None

When the interface enters cell K from the right, it has just left cell K+1, and the program must "reset" to

zero the fractional areas of certain non-intersected sides
of cell K+1.

In cases 1 through 3 the side where the interface leaves
is crossed only once by this interface. Proceeding in a clock-
wise direction from the side the interface has left, the program
resets to zero the fractional areas of the non-intersected sides
until it encounters a side that is intersected by this interface.

Case (1)
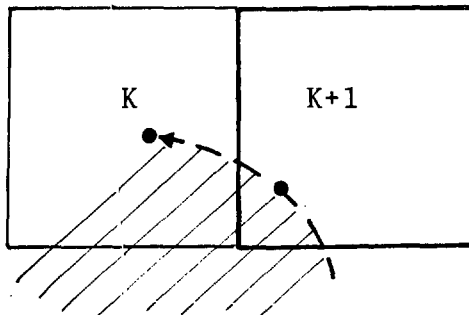
Fractional areas reset:

    Top = 0.

Right = 0.

Case (2)
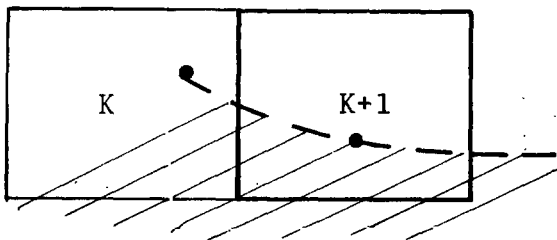
Fractional areas reset:

    Top = 0.

Case (3)
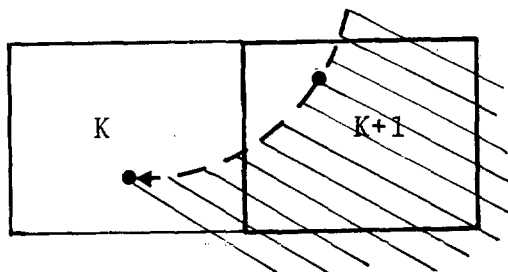
Fractional areas reset:
None

In cases 4 and 5 below, the side where the interface
leaves has been previously crossed by this interface. The
program sums the fractional areas resulting from a previous
crossing of that boundary and the current crossing. If the
sum of these areas is less than the total cell boundary area
(Case 4) none of the fractional areas of the other sides are

reset. If the sum is greater (Case 5), the program proceeds to reset to zero the areas of the other non-intersected sides of the cell.

Case (4)

Sum of areas < Total area

Previous

Previous

Fractional areas reset:

None

Case (5)

Previous

Sum of areas > Total area

Current

Fractional areas reset:

Top = 0.

Right = 0.

Bottom = 0.
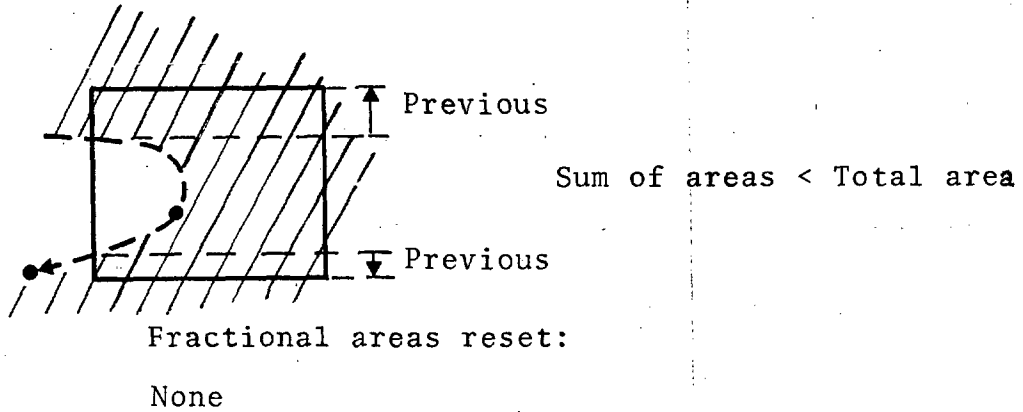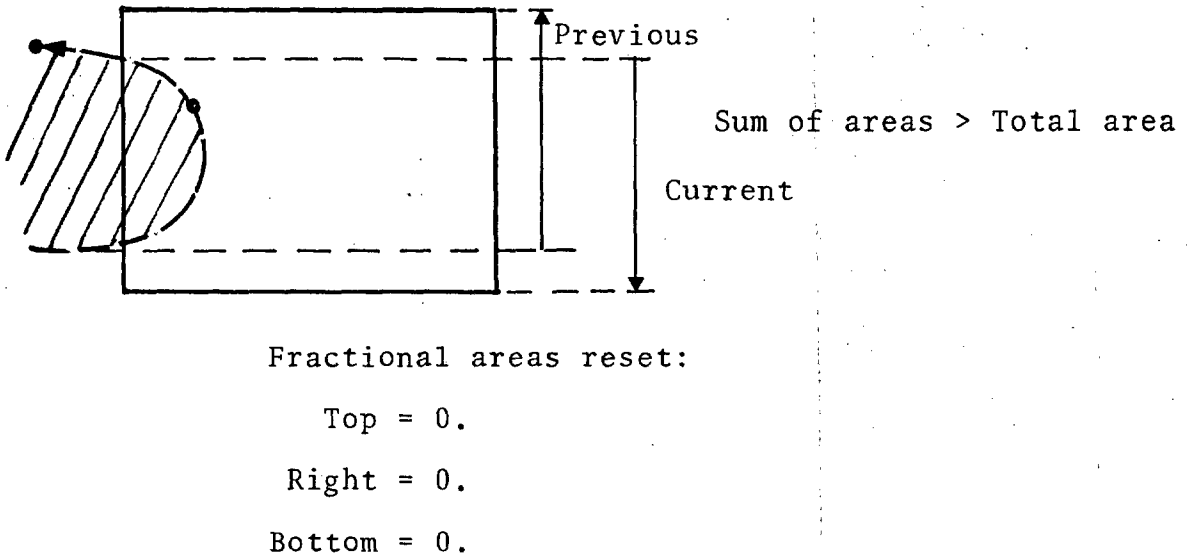
The calculation of the fractional areas is thus reduced to considering four directions of an interface with respect to a specified cell K:

1. entering cell K, leaving cell KR
2. entering cell KR, leaving cell K
3. entering cell K, leaving cell KA
4. entering cell KA, leaving cell K.
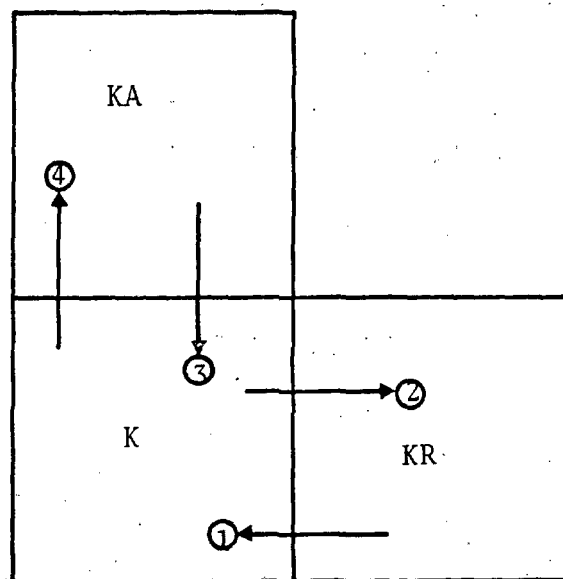
4.4 COMPUTING THE FLUX TERMS FOR INTERFACE CELLS

The computation of fractional cell boundary areas is performed in order to use the material interface positions to define the mass flux of materials across interface cell boundaries. For pure cells the mass flux between two cells is

$$\Delta m = \rho_T \, V_T \, A \, \Delta t$$

where ρ_T is the donor cell density, V_T is an average of the cell-centered velocities of the two cells, and A is the

area of the cell boundary for which the mass flux is being computed.

For cells crossed by an interface this equation becomes

$$\Delta m_N = \rho_{TN} \, V_T \, FA_N \, \Delta t/S$$

where ρ_{TN} is usually[*] the donor material N density, FA is a fractional cell boundary area for material N, and the time step, Δt, is divided by S, the number times the INFACE subroutine is subcycled. Thus an interface cell has mass flux terms for its right and top boundaries associated with each material package in the grid.

---
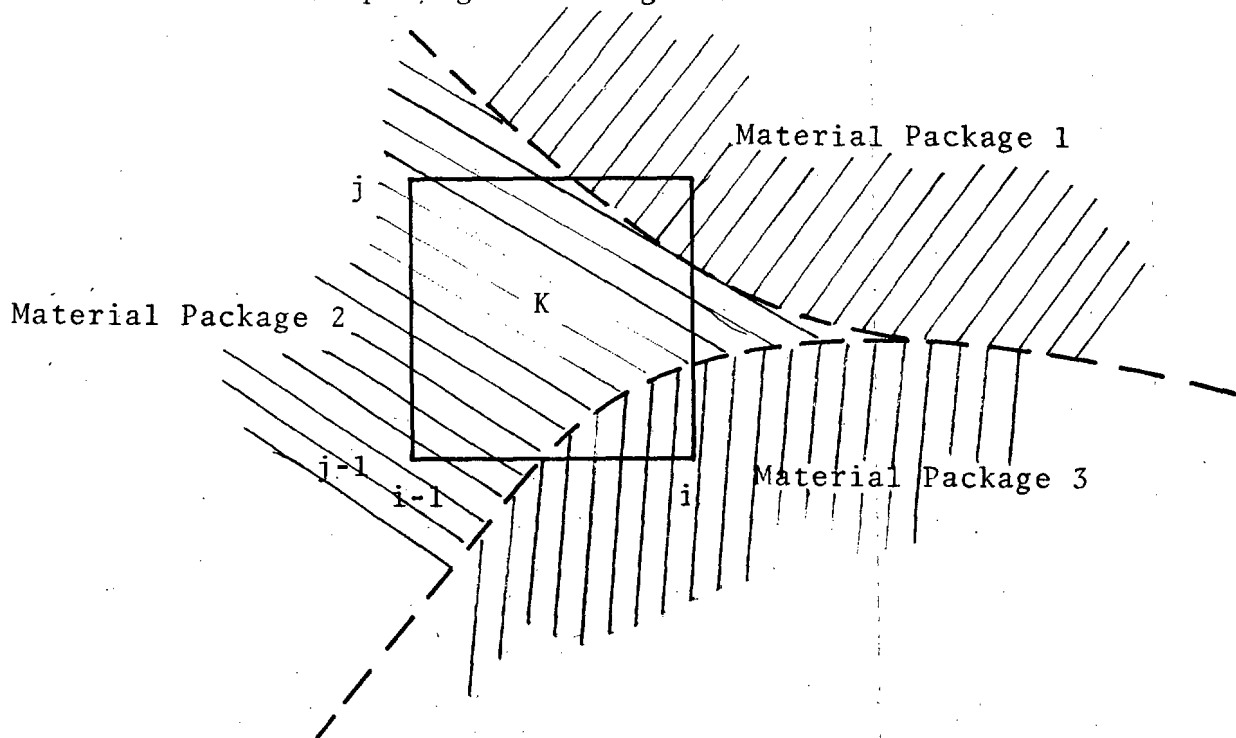
[*] For cells containing the void interface, "free surface" cells, the code applies a special rule to define the transport density, ρ_{TN}. If two neighbor cells are both free surface cells, the transport density is chosen in the usual manner according to the direction of the flow, i.e., donor material density. If only one of the two cells is a free surface cell, the density of the material in the non-free surface cell is used regardless of the direction of the flow.

In the case drawn above, assume there are three material packages in the grid. There are five non-zero mass flux terms associated with cell K. Using the FORTRAN variables and assuming M is the location in the mixed cell arrays associated with cell K, the right boundary fluxes would be

$$|SAMMP(1,M)| \sim \rho_{T1} \, U_T \; .292\left(y_j - y_{j-1}\right) 2\pi \; x_i \; \Delta t / s$$

$$|SAMMP(2,M)| \sim \rho_{T2} \, U_T \; .375\left(y_j - y_{j-1}\right) 2\pi \; x_i \; \Delta t / s$$

$$|SAMMP(3,M)| \sim \rho_{T3} \, U_T \; .333\left(y_j - y_{j-1}\right) 2\pi \; x_i \; \Delta t / s$$

and the top boundary fluxes would be

$$|SAMPY(1,M)| \sim \rho_{T1} \, V_T \; .417\left(x_i^2 - x_{i-1}^2\right) \pi \; \Delta t / s$$

$$|SAMPY(2,M)| \sim \rho_{T2} \, V_T \; .583\left(x_i^2 - x_{i-1}^2\right) \pi \; \Delta t / s$$

$$|SAMPY(3,M)| = 0.$$

The sign of these fluxes indicates the direction of the flow. The fractional top cell boundary area for the material of package three is zero. Therefore, the material of package three is not transported across the top boundary of this cell.

4.5 CREATING MIXED CELLS

4.5.1 Assigning Storage Location

Initially those cells containing a material interface are created in SETUP when the problem is generated. Later, as the interfaces move across the grid, new mixed cells are created in NEWMIX (called by INFACE). Whenever a cell boundary is straddled by a pair of consecutive tracers, INFACE checks that both cells (K and K´) on either side of the boundary are mixed. If one or both are pure (i.e., MFLAG(K) < 100,

or MFLAG(K´) < 100), subroutine NEWMIX is called and an unused[*] storage location in the mixed cell arrays is assigned to that cell. (The maximum number of mixed cells at any one time is controlled by the input variable NMXCLS which should correspond to the dimensions of the mixed cell arrays. If the user tries to generate more than NMXCLS mixed cells, NEWMIX calls for an error exit.) Before a cell K becomes mixed, its flag, MFLAG(K), indicates what material package it belongs to. NEWMIX uses that information, transferred through a variable MO in blank common, to define the mixed cell variables for the newly mixed cell, as illustrated by the following FORTRAN statements.

MO = MFLAG(K) (when pure)

(M = storage location for cell K
       in mixed cell arrays)

MFLAG(K) = M+100   (flag)

XMASS(MO,M) = AMX(K)   (mass)

SIE(MO,M) = AIX(K)   (specific internal energy)

RHO(MO,M) = AMX(K)/[TAU(I)*DY(J)] (density)

A cell that becomes "mixed" in INFACE has only one material until subroutine PH2 is executed and the other materials are transported into it. Note also that PH2 will not transport material of another package into a pure cell. The interface of the other package must first cross the cell's boundary in INFACE before the cell can receive the material of the other package in PH2.

_____

[*] A storage location M is not in use if RHO(1,M) = -1.

4.5.2 Defining Free Surface Cells

A cell containing a free surface often contains only one material and yet is still treated as a mixed cell since an interface crosses its boundaries and the fluxes across its boundaries are a function of the fractional areas computed in INFACE. Of course it is possible for a cell containing a void (i.e., a free surface) to contain more than one material. A "free surface" cell K is flagged by setting RHO(NVOID,M) = 1., where NVOID is one more than the number of material packages in the problem and M = MFLAG(K)-100.

4.5.3 Accounting for Subcycles

INFACE usually is subcycled,[*] i.e., the routine is executed several times each cycle using a fraction of the time step on each subcycle. It is therefore possible to create a mixed cell after one or more subcycles of INFACE have been completed. In that case the flux terms (SAMMP, SAMPY) for that cell have not been accumulated for the completed subcycles and need to be computed before adding in the flux terms for the current and subsequent subcycles. Therefore, when INFACE has completed at least one subcycle, NEWMIX calls FLUX after setting up the storage for the new mixed cell.

4.6 CREATING PURE CELLS

4.6.1 Defining Material that Remains in the Cell

After the fractional areas associated with each material package have been computed, INFACE searches for cells that were interface cells on the previous cycle but

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[*]Subcycling INFACE minimizes transport noise.

no longer are cut by an interface. Such a cell has become
pure and its flag, MFLAG, is made negative to signify
that fact until the transport has been completed in PH2. To
determine what material package a newly pure cell K belongs
to, INFACE first looks for a neighbor that is pure and
assumes cell K will belong to the same package as one of
its pure neighbors, e.g. in the following diagram,

Material Package 1

KL   K

Material Package 2   KB
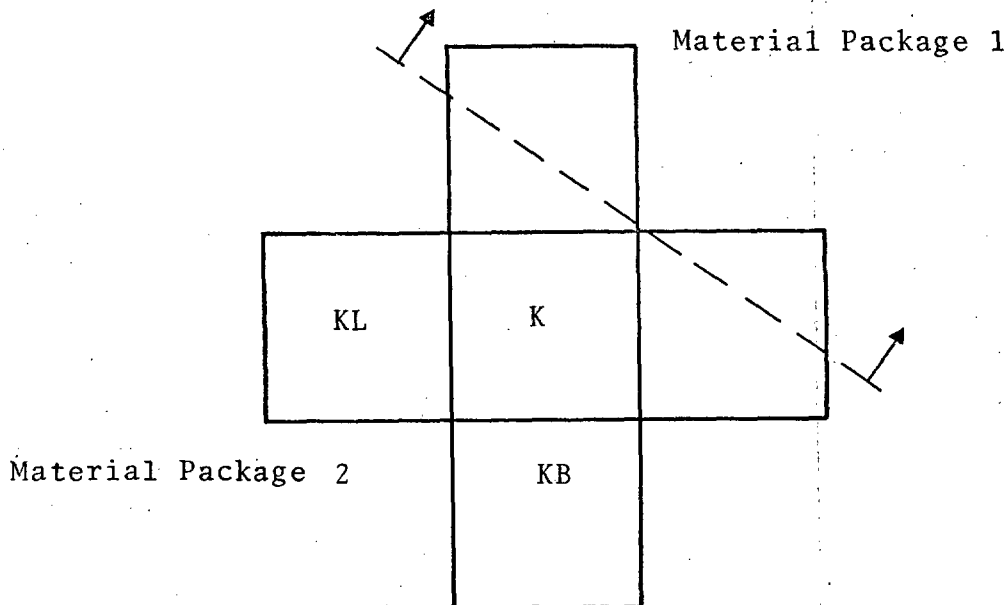
the cell below is pure (MFLAG(KB) = 2), therefore cell K
has become a pure cell belonging to package 2. In case all
four neighbors are mixed, the cells KB and KL are examined
as illustrated in the following example.
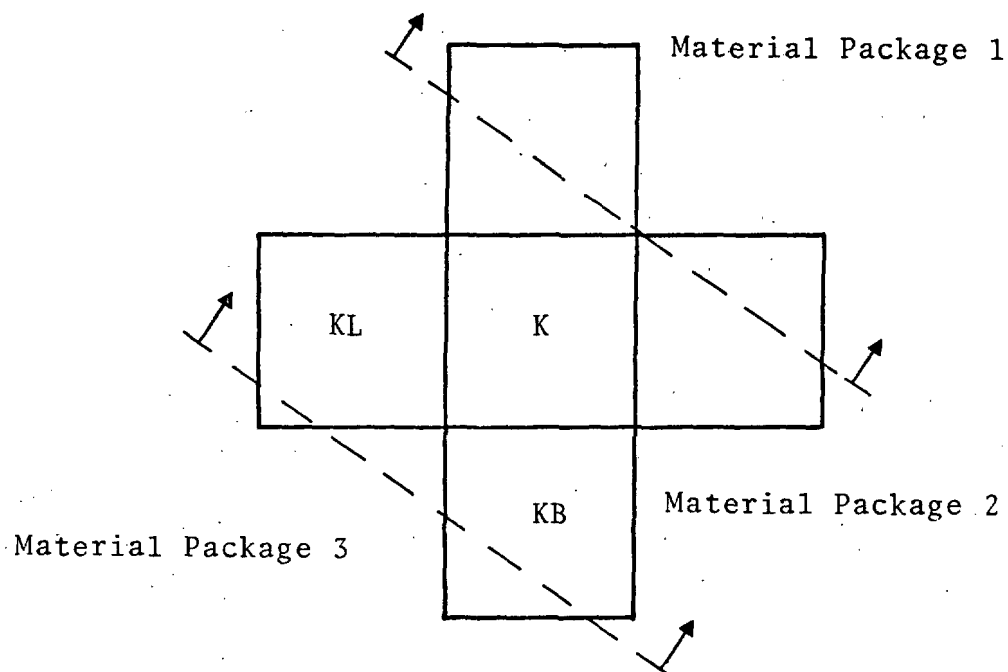
Since cell K is not crossed by an interface it follows that if the fractional area of the top of cell KB for package 2 is the total cell boundary area then cell K must belong to package 2. The cell on the left of cell K is used analogously if cell K is in the bottom row of the grid.

4.6.2 Evacuating Materials

To signify a material interface has passed out of cell K, the density of that material is set to zero. For example, consider the figure above assuming there are three material packages in the grid; the densities of cell K are redefined until the end of the cycle as follows:

$$M = MFLAG(K) - 100$$

$$RHO(1,M) = 0.$$

$$RHO(2,M) \neq 0.$$

$$RHO(3,M) = 0.$$

These zero densities are later used as a signal to adjust the fluxes of materials 1 and 3 so they will be exactly evauated from cell K. This adjustment of the fluxes usually occurs twice in INFACE, once during the first subcycle (for cells that become pure during the first subcycle) and once after all subcycles are completed (for cells that become pure after the first subcycle). For the first case the flux terms for mixed cells (SAMMP, SAMPY) are saved from the previous cycles to indicate the direction of the flow and the direction of the evacuation. In the following example t_1 represents time, and ML and MB are locations associated with the mixed cells on the left of and below cell K, respectively:
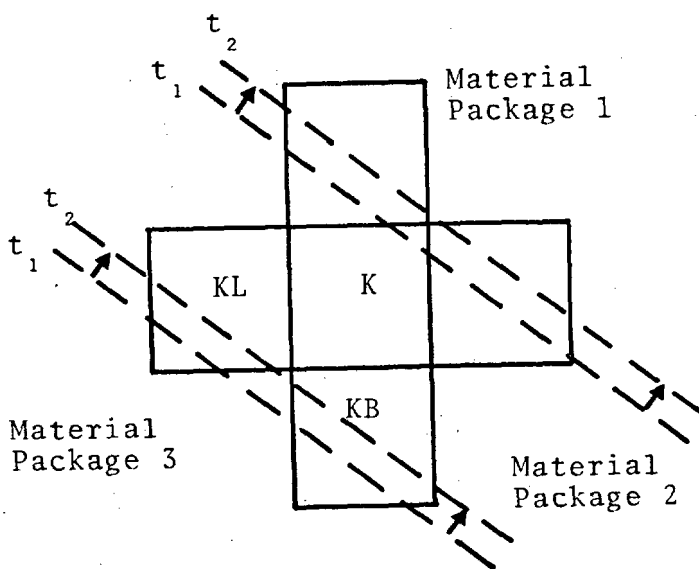
t_1 flux terms

SAMMP(1,M) > 0.

SAMPY(1,M) > 0.

SAMMP(1,ML) = 0.

SAMPY(1,MB) = 0.

At t_2, the t_1 flux terms indicate that any mass of package 1 still in cell K should be evacuated out the right and top boundaries, not the left or bottom. These evacuation procedures are used for a cell that has lost one interface but is still cut by another interface (so is still mixed) as well as for cells that have become pure.

4.7 ADVANCING TRACER PARTICLES

Each material is circumscribed by a series of massless tracer particles, which are propagated each time step a distance[*]

$$\Delta x_i = \vec{u}_i \, \Delta t$$

where $\vec{u}_i$ is a local average velocity vector for the continuum, determined by an area overlap method which gives weight to velocities in the surrounding cells. Specifically, a rectangle of cell dimensions is superimposed on the particle to be moved and then

$$\vec{u}_i = \frac{\sum\limits_{L} \vec{w}(L) \, A_L}{\sum\limits_{L} A_L}$$

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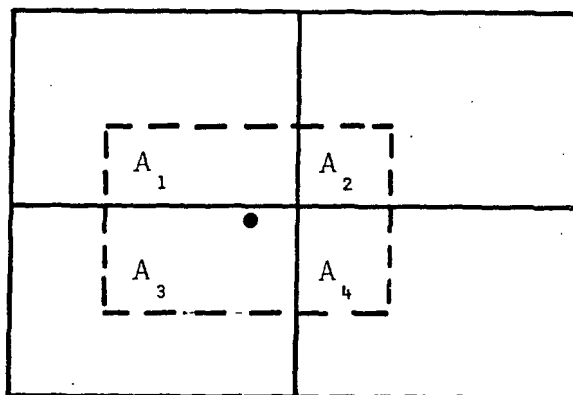
[*] Since the tracer coordinates are in grid line units, this distance is converted from centimeters to cell units.

where $\vec{w}(L)$ is the cell-centered velocity* of the overlapped cell L and A_L is the overlap area. This procedure gives a spatially continuous velocity field for particle propagation.

The cell-centered tracers (XP, YP), used only for plotting the movement of mass within the package boundaries, are moved by the same method.

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*These cell-centered velocities have been updated by PH1 (pressure effects) and PH3 (strength effects) for this time step.

V. THE PRESSURE AND DENSITY ITERATION
FOR MIXED CELLS

5.1 GENERAL METHOD

For ordinary one-material cells, it is possible to compute the pressure directly from the material density and specific internal energy. For cells containing more than one material, it is necessary to make additional assumptions or observations in order to determine cell pressure. In the present method we determine the cell pressure for mixed cells by an iteration procedure. Specifically, the densities of the various materials within the cell are varied, subject to the condition that the cell be exactly filled by the masses therein, until the individual pressures converge to a common value, taken to be the cell pressure. Specific internal energies are held constant during the iteration. This process gives, in addition to the cell pressure, the densities of the individual materials for subsequent use in the Phase 1 and Phase 3 energy partition and in the Phase 2 transport calculation.

The procedure for determining the pressure and the densities in a mixed cell is actually divided into two parts, a pre-iteration calculation to assure that the cell is filled exactly, and an iteration calculation to converge on the desired P's and ρ's.

5.2 PRE-ITERATION CALCULATION

In general, the masses and densities which exist at the beginning of the calculation do not exactly fill the cell, since cell masses were changed in the Phase 2 calculation of the previous cycle. The pre-iteration calculation fills the

cell exactly, taking account of the (computed) compressi-
bilities of the various materials in the necessary alteration
of densities. Stepwise, the code

1. Computes P_i from the old value of ρ_i
    for each material within the cell (E_i is
    assumed constant throughout).

2. Varies the ρ_i by one percent to compute
    new P_i.

3. Determines the constant-energy compressi-
    bilities, called C_i^2, for each material

    $$C_i^2 = \Delta P_i / \Delta \rho_i$$

    where the ΔP_i and $\Delta \rho_i$ are given from
    the results of (1) and (2).

4. Normalizes (see proof below) to fill cell
    exactly:

    $$\Delta V_i = -\Delta P / h_i^2$$

    where

    $$h_i^2 = \left(\rho_i \, C_i \right)^2$$

    and

    $$\Delta P = \frac{(VOL-VCELL)}{\sum \frac{m_i}{h_i^2}}$$

    $$VOL = \sum \frac{m_i}{\rho_i}$$

$$m_i = \text{mass of constituent } i.$$

$$\text{VCELL} = \text{volume of the cell.}$$

With the above increments in specific volume, then, the new specific volumes are given by

$$V_i = V_{i,old} + \Delta V_i$$

then,

$$\rho_i = 1/V_i$$

gives the desired new densities, causing the cell to be filled exactly.

It is easy to see that the calculation defined above causes the cell to be filled exactly. To show this,

$$m_i V_i = m_i \left(V_{i,old} + \Delta V_i \right)$$

$$= m_i/\rho_{i,old} - m_i \, \Delta P/h_i^2$$

$$= m_i/\rho_{i,old} - \frac{m_i (\text{VOL}-\text{VCELL})}{h_i^2 \sum \frac{m_j}{h_j^2}}$$

Summing,

$$\sum_i m_i V_i = \text{VOL} - (\text{VOL}-\text{VCELL}) \frac{\sum \frac{m_i}{h_i^2}}{\sum \frac{m_j}{h_j^2}}$$

$$= \text{VCELL}$$

showing that the new volumes $m_i V_i$ of the constituents add
up to the total cell volume, as desired.

5.3 ITERATION CALCULATION

Given $P_i = f_i(V_i, E_i)$ for materials $i = 1,M$ find
cell pressure $\overline{P} = P_1 = P_2 = \ldots = P_M$ by varying the V_i
until the P_i are equal within some specified accuracy,
subject to the conditions that the cell remain exactly
filled and that the E_i are constants.

The equation for volume conservation is

$$\sum m_i \, \Delta V_i = 0$$

and the equation that causes material i to undergo a change
in specific volume ΔV_i, such that its pressure change from
its current value $P_i^N = f_i(V_i, E_i)$ to a value $\overline{P}^{N+1}$, common
to all materials in the cell at the end of the iteration cycle,
is

$$\Delta V_i = \left(\overline{P}^{N+1} - P_i^N \right)\left(\frac{\partial V_i}{\partial P_i} \right)_{E_i} \qquad i = 1,M$$

These equations, for $i = 1 \ldots M$ can be regarded as $M+1$
equations for the $M+1$ quantities ΔV_i and $\overline{P}^{N+1}$. The
other quantities in the equations are either known constants
(m_i) or are updated each cycle of the iteration [the P_i^N and
the $(\partial V_i/\partial P_i)_{E_i}$] and are taken to be constants while solving
these equations for ΔV_i and $\overline{P}^{N+1}$. The solutions are

$$\overline{P}^{N+1} = \frac{\sum P_i^N w_i^N}{\sum w_i^n}$$

$$\Delta V_i = \frac{1}{h_i} \left(P_i^N - \overline{P}^{N+1} \right)$$

where

$$h_i = \left(\partial P_i / \partial V_i\right)_{E_i}$$

$$w_i = m_i / h_i$$

That these equations are solutions to the given equations can be verified by direct substitution.

The iteration would be exact (single cycle convergence) if the input coefficients $\left(\partial P_i / \partial V_i\right)_{E_i}$ were constants, since the solution does not involve additional approximations. In the iteration, these coefficients are refined each step of the iteration by making use of the P_i, V_i points which were generated during the previous step. Further, some pains are taken to determine realistic starting values of these coefficients, in the pre-iteration calculation, in the belief that this procedure will hasten convergence sufficiently to reduce total computing time.

Stepwise, the iteration calculation is as follows:

1. Compute new $\bar{P}^{N+1}$ and ΔV_i by the above formulas, using known values of P_i^N and $\left(\partial V_i / \partial P_i\right)_{E_i}$.

2. Compute new V_i^{N+1} by adding the above ΔV_i to the old specific volumes.

3. Compute new P_i^{N+1}, using the updated V_i.

4. Compute refined values of the coefficients $\left(\partial V_i / \partial P_i\right)_{E_i}$ by using this last point [from (2) and (3) above] and that previous P_i, V_i point (two such points are saved for each material during the iteration) which is

closest in pressure to this last point,
i.e.,

$$\left(\frac{\partial V_i}{\partial P_i}\right)^{N+1}_{E_i} = \frac{V_i^{N+1} - V_i^{'}}{P_i^{N+1} - P_i^{'}}$$

where $P_i^{'}$, $V_i^{'}$ is the closest previous point.

5. Return to (1), replacing the old values of the P_i^N and the $\left(\partial V_i/\partial P_i\right)_{E_i}$ with the updated values.

The iteration is completed when the P_i all equal $\bar{P}$ to some preset accuracy. In most experience to date, convergence has been obtained in two or three steps, where a convergence criterion $|P_i - \bar{P}| < 10^{-3}\ \bar{P}$ was used.

5.4 EQUATION OF STATE MODIFICATIONS FOR THE ITERATION PROCEDURE

The Tillotson Equation of State Formulation described in Section 2.3, has been used in most work to date with HELP and is sufficiently general to provide a satisfactory description of most media which is initially condensed and which may vaporize as a result of heating.

For pure cells, certain physically meaningless computed pressures are excluded after they are calculated, i.e., negative pressures corresponding to expansions to densities less than some input number. Also, sufficiently expanded states, for some materials, can lead to the abnormal $(dP/dV > 0)$ slope of a constant energy curve with $E < E_s$. These meaningless states are avoided by requiring, in the pure cell formulation, that the quantity $\left(A\mu + B\mu^2\right)$, where $\mu = \left(\rho/\rho_0 - 1\right)$, is replaced by its mininum value (a negative

pressure) if ρ is less than the density corresponding to this minimum value.

For mixed cells, to assure convergence of the present iteration, it is desirable to have a continuous equation of state and for $\partial P/\partial V$ to be everywhere negative. To this end the term

$$A\mu + B\mu^2$$

is replaced by

$$A\mu + h(V)B\mu^2$$

where

1. $h(V) = +1$ for $V \leq V_0$ compressed states,

2. $h(V)$ decreases linearly to -1 in the interval

$$V_0 < V < V_0 \frac{2B+A}{2B-A}$$

and

3. $h(V) = -1$ for $V \geq V_0 \frac{2B+A}{2B-A}$

The effect of these modifications is to change B to $-B$ in a continuous manner and hence insure that $\partial P/\partial V$ is always negative. Negative pressures that may result after the iteration is completed are set to zero, since tensile states in a mixed cell are assumed not to exist. In view of this latter assumption, the indicated equation of state

changes do not normally affect the physical result, since they alter only states that would ordinarily be tensile. The purpose of the changes is to make the equation of state well-behaved, so that the iteration converges properly.

VI.   SAMPLE CALCULATIONS

6.1   INTRODUCTION

HELP was written as part of a theoretical study of the armor penetration process, but is also applicable to a wide range of problems in continuum mechanics. The code, sometimes with modifications or additional routines, is now used to treat problems in gas dynamics, underground explosions, explosive-metal interactions, shaped charges, radiation hydrodynamics and a variety of impact applications. The program is especially well suited for solving problems involving extreme material distortions, in which there may be more than one material in the flow field. In the following pages, selected results are given from several problems solved with HELP.

6.2 EXPANSION OF SPHERICAL SHELLS

The purpose of this test problem was to affirm that
the code would preserve the spherical symmetry during an
explosion of a hot spherical gas bubble into a surrounding
medium of concentric shells. The test is meaningful because
the calculation is performed as a 2-D problem in axisymmetric
flow, so that any difference in treating the axial and radial
motions should appear as an asymmetry.

The initial conditions are those indicated in the
following sketch

Materials		ρ_0	γ	E_0
Package 1	Ideal Gas	.5	1.5	10^{16}
Package 2	Ideal Gas	5.	1.4	0.
Package 3	Ideal Gas	30.	1.2	0.
Package 4	Ideal Gas	10.	1.3	0.

DX = DY = 1. cm (square zones)

JMAX = 60 (rows)

IMAX = 30 (columns)
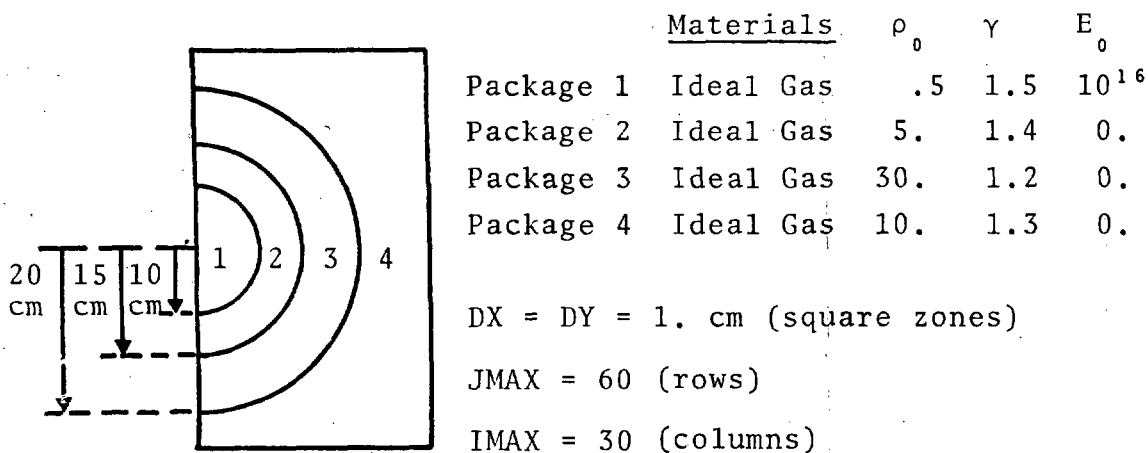
The four times, corresponding to automatic plots of the
tracer positions in Fig. 3, are 0., 0.05, 0.13, and 0.23 μsec.

As seen from these plots, the solution retains the
desired spherical symmetry to the latest times computed.
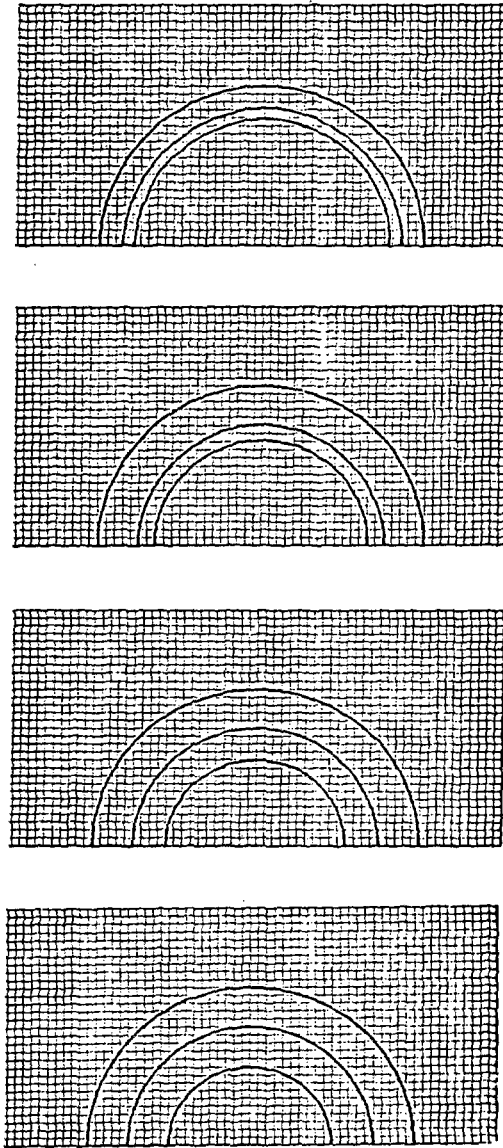
Fig. 3--Expansion of spherical shells.

6.3 IRON PROJECTILE IMPACTING AN ALUMINUM TARGET

This problem, discussed in detail in Ref. 6, is the impact of an iron projectile on an aluminum target. The impact velocity is 9×10^4 cm/sec and the shear yield strengths of the iron and aluminum, respectively, are 6 kilobars and 3 kilobars.
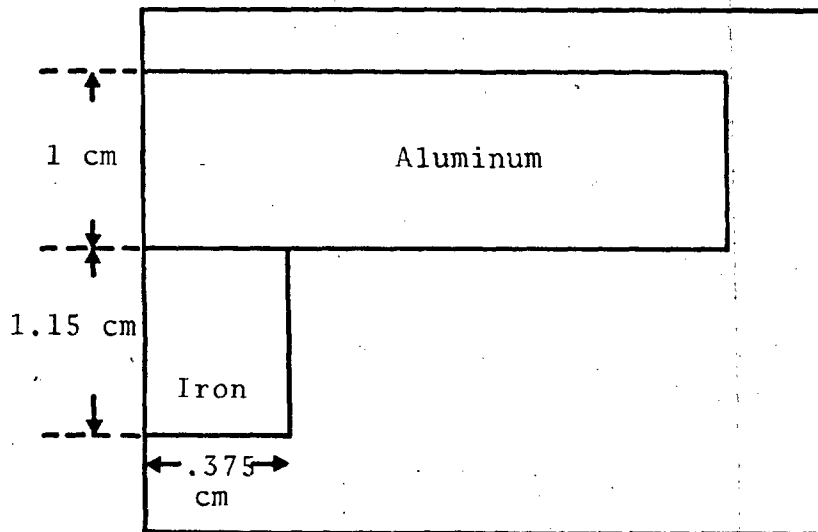
Times presented in the plots of Fig. 4 are 0, 2.5, 5.0 and 10.0 μsec. In the last frame, the 10 μsec plot is repeated with the positions of the interior (passive) tracers shown instead of the Eulerian grid lines. In all plots the outlines of the target and projectile are obtained in the plot routine by drawing straight lines between the free surface and interface tracers.
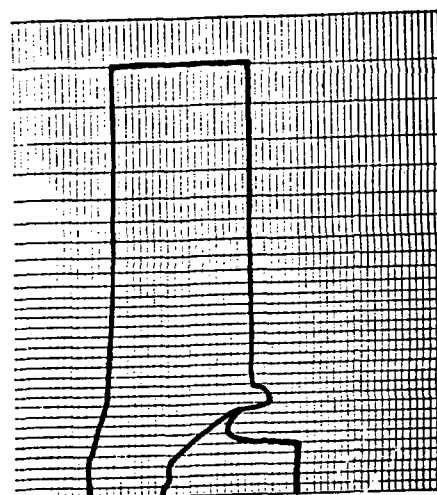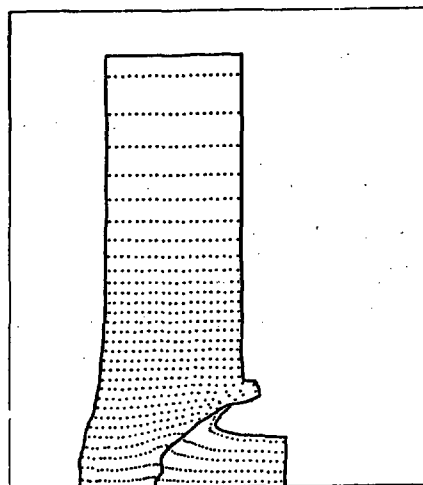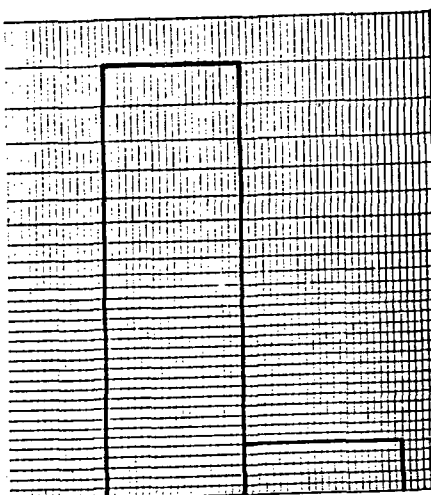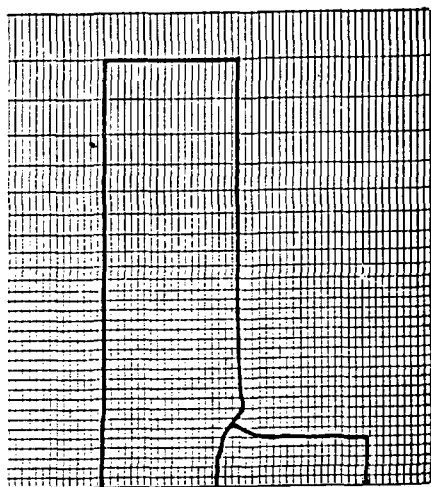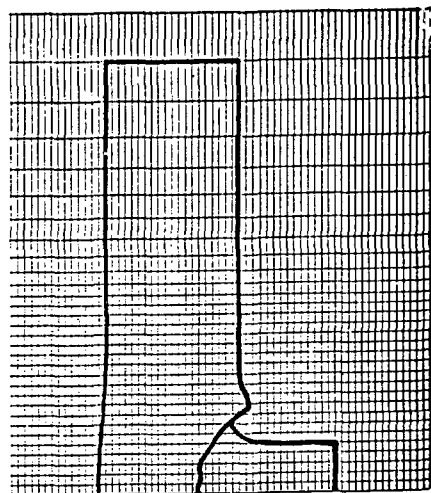
Fig. 4--Iron projectile impacting an aluminum target.

6.4   IMPACT OF A CONE ONTO WATER

To simulate an experimental study of impact into water, a conical tungsten projectile (solid near tip, thin-walled behind) impacts into water at 2.4×10^5 cm/sec.   The yield strength of the tungsten is 10.5 kilobars.   Zoning is variable (0.2 cm zones near impact center, up to 0.5 cm in far field).

Automatic plots of the configuration are given at six times in Fig. 5.   Times are 0, 2.5, 11, 18, 23 and 30 μsec. The solid tip is initially 2.2 cm in length.   As seen from these plots, the tip is substantially eroded by the penetration process.   At the latest times in the calculation the water splash is beginning to interact with the thin tungsten walls.
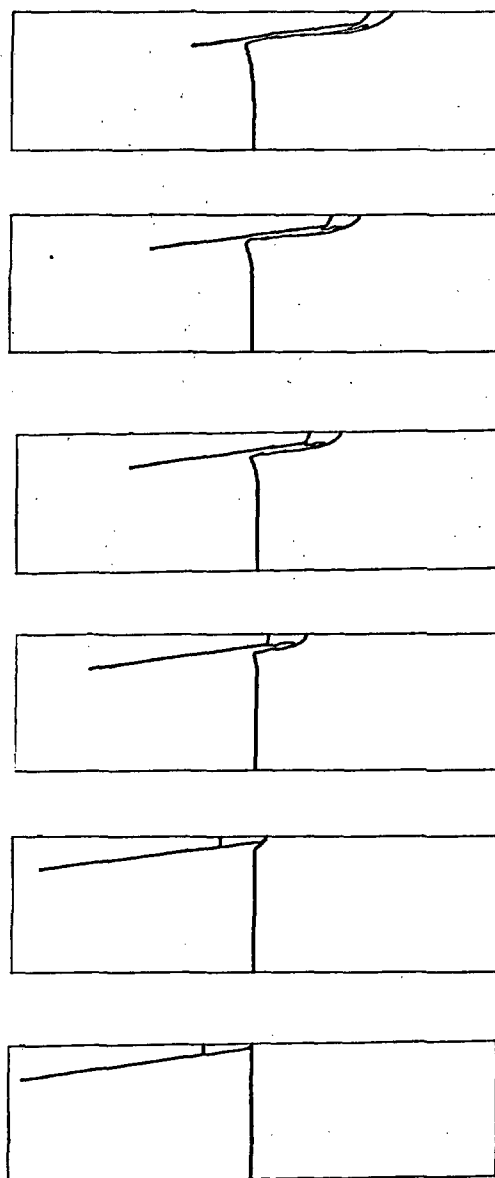
Fig. 5--Impact of cone onto water.

6.5   METEOROID IMPACT INTO LUNAR SURFACE

A stony meteoroid, 10 km in diameter, impacts the moon at 3×10^6 cm/sec. The lunar surface is taken to be basalt with 20 percent porosity.

Five times, 0, 0.4, 0.6, 0.9 and 2.5 seconds are shown in Fig. 6. At the second time, 0.4 sec, the shock wave has just reached the upper surface of the meteoroid. In subsequent plots the upper surface is seen to fold back upon the axis of symmetry, the meteoroid itself (see 0.9 sec) forming a thin layer at the bottom of the growing crater. At the later times shown, the shock wave into the moon is apparent as a region in which the tracers are relatively dense.

Detailed analysis of the output gives also the angular and velocity distribution of the debris and the regions near the center of impact in which the lunar material is vaporized or melted by the shock heating.
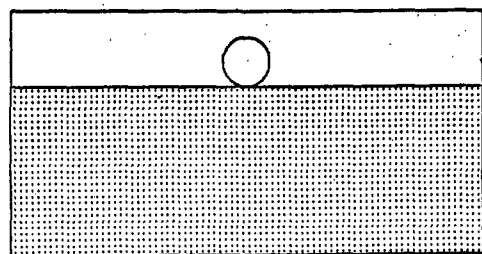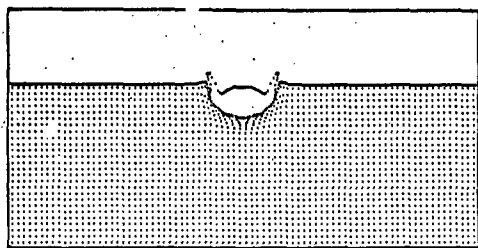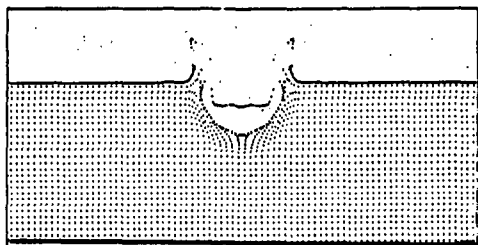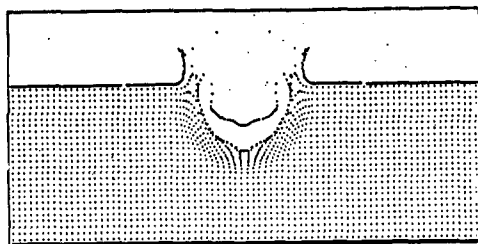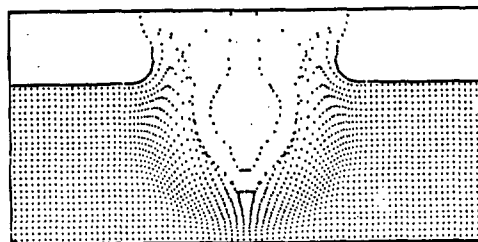
Fig. 6--Meteoroid impact into lunar surface.

6.6 JET FROM A COLLAPSING METAL LINER

A copper liner, a shortened version of that in the 105-mm shaped charge, is imploded on axis. Initial velocities are along the liner wall at 3.3×10^5 cm/sec except near the axis, where the velocity drops linearly to zero over the last 0.3 cm. The initial thickness of the liner wall is 0.27 cm and the Eulerian grid consisted of 0.05 cm square cells.

Times shown in the plots are 0, 0.7, 1.5 and 2.6 μsec. At the later times, the copper jet is seen to be well formed, with the main part of the impinging streams coalescing into the slug. This test problem is part of a general study being conducted by M. L. Gittings, R. T. Sedgwick and the present authors, for predicting the performance of shaped charges.
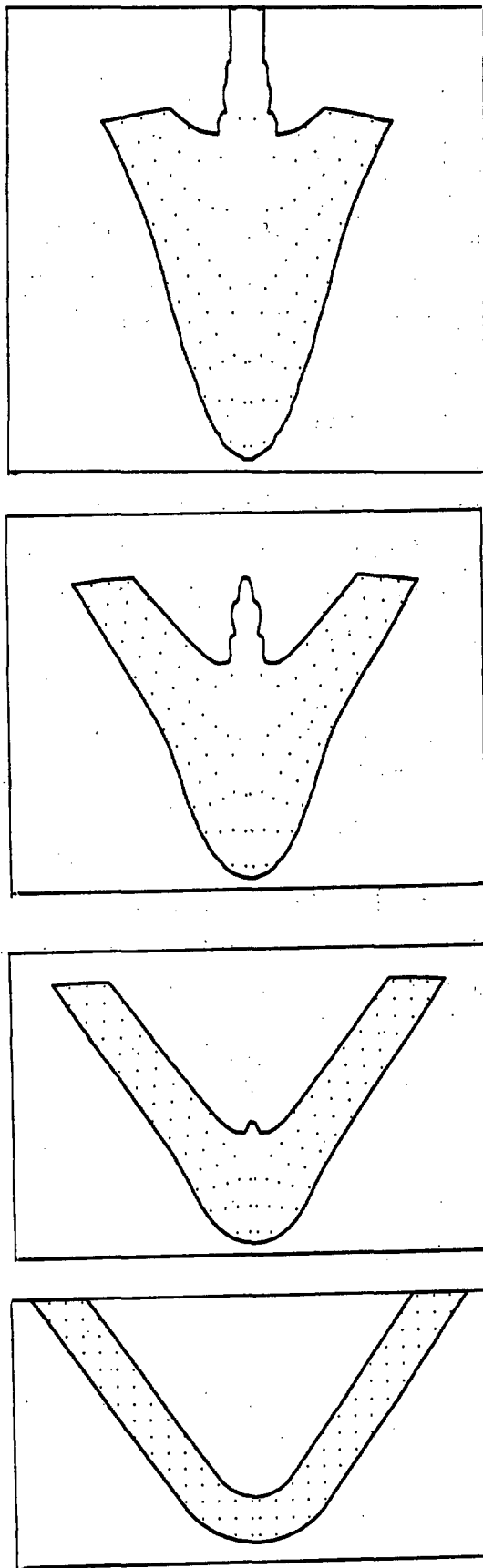
Fig. 7--Jet from a collapsing metal liner.

6.7 ARTILLERY SHELL EXPLODING IN AIR

An in-flight 105-mm-diameter artillery shell, 3000 ft/sec, is exploded in air. The shell is filled with Composition B explosive and has a 0.5-cm steel wall and a solid steel nose section.

Explosive detonation is assumed to begin at the front and proceed to the base of the projectile. Various times, 0, 30, 47, 74, 124 and 203 μsec are shown in the plots of Fig. 8, where the steel-air interfaces are plotted as asterisks and the explosive and air tracers are represented by periods. A rigid wall is present at the bottom boundary of the plots.

The air blast wave is seen to have a smooth and nearly hermispherical shape at intermediate times. At the 203 μsec time, however, the steel fragments in the forward half of the flow have nearly overtaken the air shock wave. At later times still, these fragments penetrate the main blast wave, which is decelerating due to divergence. A more detailed description of this test problem is given in a recent publication.[7]
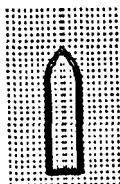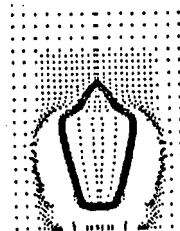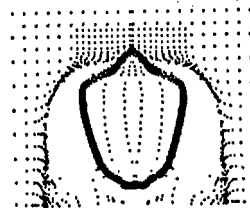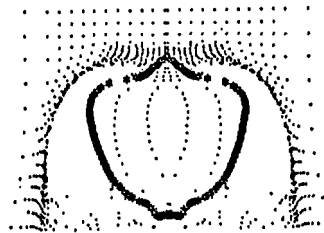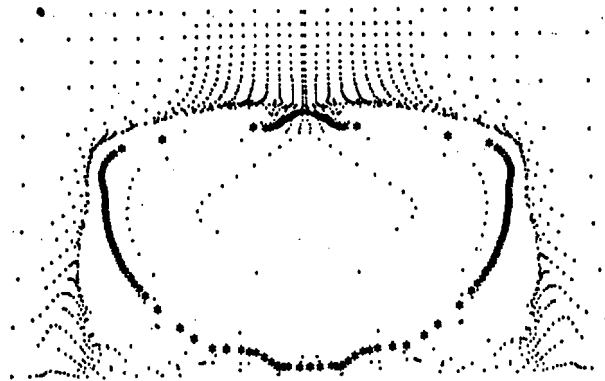
Fig. 8--Artillery shell exploding in air.

VII. GUIDE TO THE FORTRAN PROGRAM

7.1 INTRODUCTORY REMARKS

The HELP code is basically the RPM code with the added capability of treating material interfaces and of giving an elastic-plastic description of material strength. As in RPM the equations of motion are integrated in three phases, PH1, PH2 and PH3, which account for the effect of pressure, transport and shear stresses respectively. Subroutine INFACE has been added to move the material interfaces and to compute the mass flux of cells cut by one or more interfaces. An iteration to equilibrate pressures in mixed cells has been added to CDT, and EQST gives pressure as a function of density and energy for nineteen materials. The GLUE routine treats partially filled free surface cells and cells that have been over accelerated by the Phase 1 effects of pressure. These and the more peripheral routines of HELP are described in more detail in the paragraphs below. A general flow diagram of HELP is given in Fig. 9, and references to subroutines are summarized in Table 2.

7.1.1 ADDTCR

Material tracers can be added in a specified region at any point in the calculation by defining input flags (NADD, MINX, MINY, MAXX, MAXY). When NADD > 0, ADDTCR is called from EDIT on the first cycle of a run. ADDTCR adds at most the specified number (NADD) of tracers between each existing pair of tracers in the region but does not create a greater density of tracers than that initially specified by NTRACR.

MAIN ROUTINES | AUXILLARY ROUTINES

INPUT
Defines cell
quantities and
problem constants.
Reads restart
tape.

A

CARDS
SETUP
PROPRT
LOCIJ

CDT
Computes Δt.
Defines pressures.

1
Begins Cycle

EQST
P = f(ρ,E)

EDIT
Checks energy
conservation.
Writes restart
dump.
Prints.

2

MAP
ADDTCR
REZONE
COMPRS

PH1
Computes effects
of pressure terms
to update velo-
cities and inter-
nal energies.

3

GLUE

PH3
Computes effects
of deviatoric and
hoop stresses to
update velocities
and energies.

(4)
For pure hydro
calculations
PH3 is omitted.

STRNG

INFACE
Computes fluxes
of materials in
mixed cells.
Advances tracer
particles.

5

FLUX
NEWMIX
NEWRHO

PH2
Accounts for mass
flux and asso-
ciated transports
of momentum and
energy.

6
Ends Cycle

HELP
(Driver)

Fig. 9--General flow diagram of HELP program.

TABLE 2
SUBROUTINE REFERENCES

Name	Called From	Calls
ADDTCR	EDIT	--
CARDS	INPUT	--
CDT	HELP, EDIT	ERROR, EQST
COMPRS	REZONE	--
EDIT	HELP, ERROR	ADDTCR, CDT, ERROR, MAP, REZONE
EQST	CDT	--
ERROR	CDT, EDIT, INFACE, INPUT, NEWMIX, NEWRHO, PH2	EDIT
FLUX	INFACE, NEWMIX	--
GLUE	PH1	--
HELP	--	CDT, EDIT, INFACE, INPUT, PH1, PH2, PH3
INFACE	HELP	ERROR, FLUX, NEWMIX, NEWRHO
INPUT	HELP	CARDS, ERROR, SETUP
LOCIJ	SETUP	--
MAP	EDIT	--
NEWMIX	INFACE, REZONE	ERROR, FLUX
NEWRHO	INFACE	ERROR
PH1	HELP	GLUE
PH2	HELP	ERROR
PH3	HELP	STRNG
PROPRT	SETUP	--
REZONE	EDIT	COMPRS, NEWMIX
SETUP	INPUT	LOCIJ, PROPRT
STRNG	PH3	--

7.1.2 BLOCK

A block data element that defines normal density and the bulk sound speed of nineteen materials.

7.1.3 CARDS

Most of the input parameters stored in blank common are read by subroutine CARDS. The format of these input cards is described in Section 7.2.2.

7.1.4 CDT

A principal function of this routine is to compute a time step which ensures stability of the finite difference equations. This is done by finding the cell with the minimum ratio, D/ω. Here D denotes the minimum of the cell's dimensions (cm), and ω denotes the sum of the cell's maximum velocity component and its sound speed (cm/sec). For a polytropic gas the sound speed is computed as $\sqrt{\gamma p/\rho}$ and for other materials by the approximate relation $C = C_0 + \bar{B} \sqrt{P}$ where P is the pressure in the cell. The coefficient $\bar{B}$ is an input parameter and is described more fully in Section 7.3. C_0 is defined in BLOCK for nineteen materials as $\sqrt{A/\rho_0}$, where A is a coefficient in the solid equation of state. In a mixed cell the sound speed is given by a mass weighted average of the sound speeds of the materials in the cell. Each cycle CDT prints the column and row (I,J) of the cell controlling the time as well as the maximum sound speed plus velocity in the grid (MAXCUV), the maximum velocity in the grid (MAXUV) and the velocity and pressure cutoff values.

Another function of CDT is to equilibrate the pressures of materials in mixed cells using an iteration scheme that adjusts the material densities. A detailed discussion of this

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iteration method is found in Section V . The pressures
for pure cells are also updated in CDT by a call to the
equation of state subroutine, EQST, with the density, energy
and material code number of the cell (RHOW, ENERGY, N) passed
through blank common.

7.1.5 COMPRS

COMPRS is called by subroutine REZONE to create a new
cell by combining two adjacent cells into one and defining
the velocity and energy of the new cell so that energy and
momentum are conserved. If both cells are pure then the new
cell will also be pure. If one or both are mixed, the new
cell will be mixed.

7.1.6 EDIT

The periodic printing and writing on the restart tape
are executed by subroutine EDIT. The frequency of printing
and tape dumps is controlled by input parameters described in
Section 7.2.3.

On every print cycle EDIT prints the mass, total energy,
internal energy, kinetic energy, axial and radial momentum,
and plastic work for each material package as well as for the
entire grid. The changes in energy due to evaporation and
losses out boundaries are also accounted for. The coordinates
of the material tracers circumscribing each package are printed
in cell units. Summary graphs of the compression, pressure,
velocities and internal energies are printed if the input
parameter, MAPS, is non-zero. And finally, the pressure, velo-
cities, internal energy, compression and stress deviators for
all cells in the active grid are displayed on "long" EDIT prints
and for each cell in the axis column on "short" EDIT print.

Another important function of EDIT is to compute the relative error in the total energy of the grid when summed over all the cells, and to check that this error does not exceed a limit specified by the input variable DMIN.

EDIT also senses when execution should stop and sets the exit flag, WFLAGL.

7.1.7 EQST

The equation of state constants for nineteen materials are stored in DATA statements in EQST. (These constants are displayed in Table 1.) Given a code number for specifying the material, a density and an energy, EQST computes a pressure. A more detailed discussion of the equation of state is given in Section 2.3. EQST is called by CDT to compute pressures of pure cells as well as of materials in mixed cells.

7.1.8 ERROR

This subroutine is called in the case of certain error conditions tested on by the code, e.g., the energy sum error exceeding the specified limit. ERROR prints a message identifying the general location of the error condition, lists the Z-block variables (first 150 words of blank common), calls EDIT to do a long print and a tape dump, and then calls exit.

7.1.9 FLUX

The mass flux across the right and top boundaries of a mixed cell for each material in that cell is computed in FLUX. These fluxes are stored in the SAMPY and SAMMP arrays and executed in subroutine PH2 which does the actual transport of material across all cell boundaries. FLUX uses the position

of the material interfaces to compute the fluxes of the
various materials in the cell. See Section 4.2
for a more complete description of transport from mixed
cells.

7.1.10 GLUE

Free surface cells, or cells of very small mass, can
be over-accelerated by the pressure forces (PH1) or the
strength forces (PH3). See the discussion in Section 3.5.
GLUE is called by PH1 after the effects of pressure have
been computed and after PH1 searches for cells with un-
realistic velocities and makes the flags (MFLAG) of these
cells negative.

GLUE combines or "glues" a cell with a negative flag
or a free surface cell to its adjacent neighbor with the
highest pressure. For each velocity component, GLUE computes
a new velocity for the two cells which conserves momentum.
The new specific internal energy for the cells is then
chosen to conserve total energy. The overall effect is to
give the two cells the same velocity and specific internal
energies. The negative flags are made positive as the
cells are processed by GLUE.

7.1.11 HELP

The overall cycling of the calculation is controlled
by HELP, as shown in Fig. 9. It is the main routine of the
code and calls INPUT, CDT, EDIT, PH1, PH3, INFACE and PH2, in
that order. When needing to see the effects of each phase
of the calculational cycle, HELP will respond to the input
parameter INTER and call EDIT on print cycles after PH1 and
PH3 as well as after CDT. HELP also calls EXIT on the normal
cessation of the calculation.

7.1.12 INFACE

The tracer particles that circumscribe each material package are moved in INFACE. These particles when connected by straight line segments are used by INFACE to determine where the interfaces cut cell boundaries and to compute the flux of materials across these partial cell boundaries. For a given cycle, INFACE senses which cells become mixed and which become pure. Section VI gives a detailed description of the logic and conventions employed by INFACE.

7.1.13 INPUT

Instructions for running problems are interpreted by INPUT which can either start or restart a calculation. It calls SETUP and CARDS, as necessary, to prescribe the initial conditions and to read the input deck. It can also read a HLPCLM tape for problems whose initial conditions cannot be described by SETUP.

7.1.14 LOCIJ

This routine is used to identify the column or row location of a tracer point whose coordinates are in centimeter units. SETUP calls LOCIJ when converting the tracer coordinates to cell units for the void package as well as when defining the boundaries of rectangular packages.

7.1.15 MAP

This subroutine is called by EDIT when the input parameter MAPS is non-zero. It displays the properties of each cell in the active grid using an alphabetic scale. [On cycle 0 it presents the entire (IMAX by JMAX) grid.] One obtains contour maps of the compression, pressure, radial and

axial velocities, and internal energy. Since the compression of mixed cells is not computed, asterisks are displayed for mixed cells in the compression map. The scale of each map is adjusted according to the current minimum and maximum values of each property.

7.1.16 NEWMIX

When a pure cell K becomes mixed, NEWMIX assigns to it a location M in the mixed cell arrays (SIE, XMASS, RHO, etc.) The flag of cell K is redefined so as to associate the K and the M indices (MFLAG(K) = M + 100). The fact that MFLAG(K) is greater than 100 indicates cell K is a mixed cell. Otherwise, if cell K were pure, MFLAG(K) would be equal to the number of the package that contained cell K.

NEWMIX is called from INFACE which is usually subcycled. If cell K becomes mixed after the first subcycle of INFACE, NEWMIX calls FLUX to bring the definition of the flux variables for cell K to the current time. (See also Section 4.5 for further discussion of mixed cells.)

7.1.17 NEWRHO

When the material N interface first enters a cell, INFACE calls subroutine NEWRHO to define the density of material N for cell K by looking at the density of material N in the neighboring cells closest to the interface. (This density value, stored in the RHO array, is updated in CDT when pressures are equilibrated at the beginning of each cycle.)

7.1.18 PH1

The effect of the pressure gradient in updating the velocities and the internal energy is computed here. (The

numerical method is described in detail in Section 2.2.2.)
Cells with small masses that are over-accelerated by these
pressure effects are flagged, and subroutine GLUE is called
to give more realistic velocities to these cells.

7.1.19 PH2

Mass transport and the associated flux of momentum and
energy are accounted for in PH2. (The numerical method is
described in detail in Section 2.2.4.) After the transport
has been completed, PH2 corrects for negative mass quantities
in mixed cells. Before printing the symbolic map which dis-
plays the material package numbers and mixed cell locations,
PH2 redefines the flags of cells that have become pure.

7.1.20 PH3

The deviator stresses acting on each cell edge and the
hoop stress are determined in PH3 and the resulting velocity
and internal energy increments are computed. (Details are
given in Section 2.2.3.) PH3 calls STRNG to compute the
strength of the material in a cell. PH3 can be bypassed by
setting CYCPH3 = -1., when the effects of strength are omitted.

7.1.21 PROPRT

Subroutine PROPRT is called by SETUP to define the flag
and mixed cell variables for a cell on the edge of a rectan-
gular package.

7.1.22 REZONE

To add more material and diminish the number of zones
in the active grid the REZONE routine can lump two cells into
one or four cells into one, and add columns and rows to main-
tain an IMAX by JMAX grid. The material in the rows (or

columns) to be added will have the same properties as the last row (or column) of the grid. Therefore a package that extends to the edge of the grid in the x-direction before rezone is assumed to be infinite in that direction. Likewise for the y-direction. The momentum and total energy of the original grid are conserved when the cells are combined by converting some kinetic energy into internal in a process loosely called "thermalizing." The DX, X, TAU and TX arrays are redefined when the grid is rezoned in the x-direction. For the y-direction rezone, DY, Y and TY arrays are redefined.

All the cell quantity arrays (AMX, AIX, U, V, P, MFLAG, STRSZZ, STRSRR, STRSRZ) are redefined when rezoning in one or both directions. This REZONE routine is very limited and should be used only to add more void or more material to a semi-infinite package. All finite boundaries of a package must be within the original grid.

REZONE is called from EDIT when certain input parameters are set (NUMREZ, REZFCT, SS4). It can be "forced" on the first cycle of a restart run by setting SS4=1. Or it can be "triggered" by PH2 which senses when a finite signal reaches the edge of the grid.

7.1.23 SETUP

This routine generates problems involving one spherical package and/or several cylindrical packages. (The "CLAM"[2] generator has been modified to set up HELP problems that involve more complex geometries.) SETUP also defines the X, Y, DX, DY and TAU arrays for variable as well as constant zoned grids. The properties of each cell in a material package are determined by the input cards read by SETUP. The coordinates of the material tracers are defined as each package is generated.

Problems which generate a void package require additional
input cards to define the "free surface" tracers. However,
when the void surrounds only a spherical package, these
cards are unnecessary as the free surface tracers are de-
fined when the sphere is the first package generated. Both
SETUP and the "CLAM" generator move the tracers very slightly
off of the grid lines to simplify the logic of INFACE. The
input cards read by SETUP are described in Section 7.2.3.

7.1.24 STRNG

The yield strength of the material in a cell is computed
by STRNG. (The strength constants for each material are de-
fined by input cards when the problem is generated.) The
strength of material in a mixed cell is a volume weighted
average of the strength of the materials in the cell. STRNG
is called from PH3.

7.2 GENERATING PROBLEMS

7.2.1 General Capabilities and Limitations

The HELP problem generator can set up one sphere and any number* of cylindrical** packages or subpackages with the following restrictions.

1. All cells within a package will have the same properties.

2. The right and top grid boundaries cannot serve as package boundaries. If a package extends to the right edge of the grid the package is assumed to be infinite in the x-direction. Likewise for the top edge of the grid.

3. The grid should have at least three columns. (Special conditions for 1-D problems have not been included in the code.)

4. The right and top grid boundaries are transmittive although the bottom boundary can be either reflective or transmittive.

5. Void opening and closing mechanisms have not been generalized and included in the current version of HELP.

6. The code defines only one γ for the ideal gas equation of state. Problems with more than one ideal gas require code changes. It might be noted here also that the

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*Limited only by the dimensions of the mixed cell arrays.

**In plane coordinates circular and rectangular packages are generated.

essentially infinite density obtained by
iterating for pressure equilibration in
a cell containing cold (E = 0) and hot
gas may lead to spurious mass transports.
More gradual energy changes are necessary
for gases.

7. The center of a sphere package must be
on a horizontal grid line and on the
axis of symmetry. Furthermore, if the
sphere boundary is a free surface, the
sphere must be the first package
generated.

8. The DATA statements in BLOCK and EQST must
be changed if the user wishes to redefine
the normal density or the equation of state
constants of a material.

The following capabilities of HELP should also
be noted here:

1. The number of material packages generated
is limited only by the dimensions of the
variables, which can be increased up to
the limit of the user's computer core
storage.

2. A cell can contain any number of materials
and material interfaces.

3. Two packages can actually contain the
same material (use the same equation of
state) but have different initial velo-
cities, energies or densities.

4. A package can be divided into any number
   of spatially disconnected subpackages. (See
   definition of MAT in the following discussion
   of input variables.)

5. When strength effects are unimportant, the
   PH3 subroutine can be bypassed by setting
   to -1. the input variable CYCPH3. In this
   case the code is "hydrodynamic" in that it
   solves problems in compressible fluid dynamics.

7.2.2 Format of Input Cards

A HELP problem is generated by three kinds of input
data:

1. The identification header card read by INPUT
   that contains alphanumeric symbols in columns
   2 through 72.

2. The input data read by subroutine CARDS that
   defines variables in the Z-block (the first
   150 words of blank common).

3. The input data read by subroutine SETUP
   that defines the DX, DY arrays when either
   or both are variable, the boundaries and
   properties of the material packages, and
   the coordinates of certain free surface
   tracer particles.

The input cards read by CARDS use the following format:

(I1)	(I5)	(I1)	(E9.4)	(E9.4)
Col. 1	Cols. 2-6	Col. 7	Cols. 8-16	Cols. 17-35 etc.
L	M	N	Z(M)	Z(M+1)

The value of L for each input variable is listed in the discussion that follows. When L = 0 the variable defined is typed real. When L = 2, the variable defined is typed integer. When L = 1 the variable defined is typed real and the card is the last one to be read until CARDS is called again. The value of M is the location in blank common of the variable being defined in Cols. 8-16; this location number is also listed below for each input variable. The value of N indicates how many consecutively stored variables of the same type are defined by a card (usually N = 1). Z(M) must be punched with an E format even when it is defining an integer variable.

The format of the cards read by SETUP varies and is described in the following discussion of the input variables.

7.2.3 Description of Input Variables

The following list of input variables is ordered as the HELP setup input deck should be ordered. The first group of cards are read by CARDS, the second group by SETUP and the last dummy card is read by CARDS. Many cards in the first group can be omitted when the user is inputting a zero value.

Input Read by CARDS

Variable Name	Column One Flag	Location in Blank Common	Definition
PK(1)	1	151	Problem number. Any number from .0001 to 99.9999. Should be identical to PROB. This is the second input card of a restart as well as a setup deck.
PROB		1	Problem number.
NFRELP	2	5	Gives frequency of "long" EDIT prints which gives velocities, energy, compression and stresses for all cells in active grid. A "short" EDIT gives this information only for the axis column. When NFRELP=2, every second EDIT print is "long".
NDUMP7	2	6	Gives frequency of restart tape dumps. Program will dump only when EDIT prints. If NDUMP7=5, a restart dump will be made every 5th time EDIT prints.
ICSTOP	2	7	Gives cycle at which execution will stop if the user wants to specify a cycle rather than a time on which to stop. If stopping on time omit this card.
NUMREZ	2	12	The total number of times the grid will be rezoned. If not rezoning the grid omit this card.
KUNITR	2	14	The tape unit INPUT will read from. KUNITR will be set equal to 7 unless defined by this input card. If restarting from a HLPCLM tape, KUNITR must be 7.
IPR	2	15	Maximum number of iterations CDT will perform when attempting to equilibrate the pressures of materials in a mixed cell. If the pressures are not within an epsilon (PRCNT) of the average pressure after IPR iterations, an error exit occurs. (Try 35.)
PRCNT		16	Convergence minimum for the pressure equilibration of materials in a mixed cell. All pressures, P_i, must satisfy the following: $\left\|(P_{av} - P_i)/P_{av}\right\| <$ PRCNT. (Try .001.)
KUNITW	2	17	The tape unit EDIT and SETUP will write on. KUNITW will be set equal to 7 unless defined by this input card. If restarting from a HLPCLM tape this tape unit should not be 7 unless the user wants SETUP to write over the HLPCLM dump.
IGM	2	21	When IGM = 0. code uses cylindrical coordinates. When IGM = 1., code uses plane coordinates.
DMIN		24	The maximum relative error in the energy sum that the user wants to tolerate. The error is computed as follows: $$\left\|\left(\sum_{K=1}^{KMAX} E_k - ETH\right)/ETH\right\|$$ where E_k is total energy of cell K and ETH is theoretical energy in the grid--computed in SETUP and updated in PH1, PH2, PH3. Usually DMIN $= 10^{-3}$.

Variable Name	Column One Flag	Location in Blank Common	Definition
CVIS		27	A flag that describes the boundary condition at the bottom of the grid. If CVIS = 0., the boundary will be reflective. If CVIS = -1., the boundary will be transmittive.
IMAX	2	33	The number of columns in the grid. If planning to rezone the grid once IMAX should be an even number; IMAX, for this version of the program, should be at least 3.
JMAX	2	35	The number of rows in the grid. (Follow same rules as for IMAX if grid is to be rezoned.)
MAPS	2	42	When MAPS > 0., part of the EDIT print is a set of symbolic maps of the compression, pressures, velocities, and specified internal energies of cells in the active grid.
NUMSCA	2	43	The number of times the cycle or time interval between EDIT prints is increased. (See PRLIM)
PRLIM		44	The time or cycle at which the EDIT print frequency is changed. Example: you wish to print every 10^{-8} sec until T = 10^{-7} sec; and every 10^{-7} sec until T = 10^{-6} sec, and every 10^{-6} sec thereafter. Set: PRDELT = 10^{-8} IPCYCL = 0. PRLIM = 10^{-7} PRFACT = 10. NUMSCA = 2. NOTE: When PRDELT is increased by a factor, PRLIM is also increased by the same factor for the next rescaling. If you want a constant print interval omit NUMSCA, PRLIM, PRFACT.
PRDELT		45	The time (sec) between EDIT prints. If printing on cycle intervals omit this card.
PRFACT		46	The factor by which the print interval (PRDELT or IPCYCL) and the print limit (PRLIM) are increased. (See PRLIM)
I1	2	47	The number of columns in the grid that have non-zero velocities or energies plus 2. I1 and I2 define the "active" grid. (Most calculations are done only inside the active grid.)
I2	2	48	The number of rows in the grid that have non-zero velocities or energy plus 2.
IPCYCL	2	49	The number of cycles between EDIT prints. If printing on time intervals omit this card.
IVARDY	2	54	When the grid is variably zoned in the y-direction (DY's not constant), IVARDY = 1. When the grid has constant zoning in the y-direction, this card is omitted. (IVARDY = 0.)

Variable Name	Column One Flag	Location in Blank Common	Definition
N6	2	56	This parameter causes the program to set negative pressures to zero in the rows of the grid below J = N6. Omit this card if you want to allow negative pressures everywhere in the grid. If you want to set negative pressures to zero everywhere, set N6 = 10⁴.
GAMMA	--	62	(a+1.) in gamma-law equation of state. Used only for an ideal gas.
NSIDES	2	67	The number of line segments that describe the void package not counting the grid boundary lines. Omit this card if there is no void or if the only free surface is a spherical surface.
NMAT	2	68	The number of material packages, excluding the void package. The maximum number of packages is determined by the dimensions of the mixed cell variables in MCXELL COMMON and in BLANK COMMON. The minimum number is two.
CYCMX		69	The number of passes through subroutine INFACE in one time step to minimize transport noise near interfaces. Usually CYCMX = 2, but when calculation costs permit, CYCMX = 8 is a good maximum.
CYCPH3		70	The number of passes through subroutine PH3 (strength phase) in one time step. Usually CYCPH3 = 1., but larger values can help to dampen oscillations. When the calculation is pure hydro (no strength effects) set CYCPH3 = -1.
REZFCT		71	A rezone flag. When the grid is to be rezoned at some point in the calculation, REZFCT = 1. Omit this card only if the grid will never be rezoned.
NTRACR	2	72	The number of tracer particles per cell edge to be used to circumscribe the material packages. Include this card even when the problem is generated by HLPCLM, since this parameter is used in adding tracers (ADDTCR) as well as in setting up material packages (SETUP). (Try 2 to 5.)
NMXCLS	2	73	The maximum number of mixed cells to exist in the grid at any one time during the calculation. Consult the dimensions of the mixed cell arrays in MXCELL COMMON and BLANK COMMON.
NTPMX	2	78	The maximum number of tracer particles per material package. This maximum should depend upon the dimensions of the TX and TY arrays.
GLUED		79	When GLUED = 1. all mixed cells are "glued" after PH1 updates velocities and energies due to pressure gradients. This card is usually omitted since the program will automatically glue underdense cells that are "kicked" by PH1. This option has been used when all the mixed cells were between an expanding hot gas and a cold dense material.
NTCC	2	81	When NTCC = 1., passive cell centered tracer particles are initially placed in the center of every fourth non-empty cell and subsequently moved across the grid with the material. These are used only for producing particle plots. When this card is omitted (NTCC = 0), cell centered tracers are not computed.

Variable Name	Column One Flag	Location in Blank Common	Definition
IVARDX	2	83	When the grid is variably zoned in the x-direction (DX's not constant), IVARDX = 1. When the grid has constant zoning in the x-direction, this card is omitted (IVARDX = 0.).
EMIN		85	The minimum specific internal energy of an ideal gas to be used in the pressure iteration. Usually EMIN = 10^7.
PMIN		86	A pressure cutoff for the zero cycle of the calculation. Thereafter PMIN is computed in CDT. Usually PMIN = 10^6.
INTER	2	87	A special editing flag for debugging. When INTER = 1., EDIT prints after PH1 and PH3 as well as after CDT on print cycles. When INTER = 99., PH3 prints energy totals after updating each cell; PH2 does the same when INTER = 7. The later options are in addition to the extra EDIT prints and involve many pages of output.
NODUMP	2	96	When NODUMP = 1., EDIT will not write any tape dumps. (This flag overrides the NDUMP7 option, but does not prevent SETUP from writing the cycle 0 dump.)
ROEPS		110	A round-off epsilon used to define cutoffs in the calculation. Try ROEPS = 10^{-5}.
FINAL		113	The final value of the stability fraction used in determining the time step. Usually FINAL = .4. (See STAB)
NADD	2	118	A flag for adding tracer particles. If NADD = 3, EDIT calls ADDTCR on the first cycle of the run. ADDTCR adds 3 tracers between all existing pairs of tracers in a specified region (except where there will be more than NTRACR tracers in a cell.) NADD is reset to zero in ADDTCR.
MINX	2	119	These parameters specify the left and right columns and the bottom and top rows, respectively, of the region in which ADDTCR will add tracer particles. This card must be included when NADD ≠ 0.
MAXX	2	120	
MINY	2	121	
MAXY	2	122	
IEXTX	2	123	A rezone flag. The grid will be rezoned in the x-direction only when IEXTX = 1. Omit this card if the grid is not to be rezoned or if it is to be rezoned in the y-direction only.
JEXTY	2	124	A rezone flag. The grid will be rezoned in the y-direction only when JEXTY = 1. Omit this card if the grid is not to be rezoned or if it is to be rezoned in the x-direction only.
SS4		130	A rezone flag. The grid is rezoned on the first cycle of the run when SS4 = 1. (This overrides REZFCT but not NUMREZ which must be non-zero for REZONE to be called.)
DXF		136	The x-dimension of cells (cm) when the grid has <u>constant</u> zoning in the x-direction. Omit this card if the x-dimension of cells varies.
DYF	--	137	The y-dimension of cells (cm) when the grid had <u>constant</u> zoning in the y-direction. Omit this card if the y-dimension of cells varies.
STAB	--	139	The initial value of the "stability fraction" for the calculation of DT. If FINAL=0., STAB is constant. Otherwise its value increases from the input to FINAL in a geometric progression.

Variable Name	Column One Flag	Location in Blank Common	Definition
DTMIN		144	The minimum value for the time step. The program gives an error exit if CDT calculates a Δt < DTMIN. (Try DTMIN = 10^{-11})
JCENTR	2	147	The J-line that passes through the center of the sphere package. (Used in GLUE and PH2 to insure symmetric results in spherical calculations.) Omit this card if the problem does not involve a spherical package.
RADIUS		148	Radius (cm) of spherical package. Omit this card if the problem does not involve a spherical package.
BBAR		149	A constant used in calculation of local sound speed of all materials except ideal gases. $C = C_o$ + BBAR $\cdot$ $(\sqrt{P})$. (Usually BBAR = 0.5)
TSTOP	1	50	The time (sec) at which the calculation will stop. This card (with a "1" in column one) should not be omitted even when stopping on a specified cycle, and should always be the last card of a restart deck.

(Insert all cards described on the following pages)

EMOB	1	150	Dummy end card of SETUP deck. Omit from restart deck. EMOB = 0.

Input Read by SETUP

1. Defining Cell Dimensions <u>When Variable</u>. User can specify variable DY and /or DX values. In the case that both are variable, the <u>DY array is defined first</u>. Both arrays are read using the following format:

Variable Name	Columns	Definition
NT(I), I=1,4	1-4 5-8 9-12 13-16	NT(I) is the number of zones that have dimension TEMP(I). When all DY's are defined, set next NT = 999. Likewise for DX's.
TEMP(I), I=1,4	17-30 31-40 41-50 51-60	NOTE: When DY's are variable, exactly JMAX DY's must be defined. Likewise, exactly IMAX DX's must be defined.

2. Defining Each Material Package (4 cards for each package or subpackage).

Variable Name	Card Number	Columns	Definition
IGEOM	1	1	Defines package geometry. IGEOM = 1 for a rectangle. IGEOM = 2 for a sphere (I1).
MAT(N)	2	1-6	Material code number of this package (e.g., if |MAT(1)| = 3 then package one is iron. See list in EQST). If material package N is divided into several spatially disconnected subpackages, MAT(N) is negative for all but the last subpackage (I6).
PACRT PACTP PACLF PACBT	2	7-16 17-26 27-36 37-46	(Omit for spherical package.) Location (cm) of right, top, left, bottom boundaries, respecitvely, of rectangular package. (4E10.4) Set PACRT = -1. if package is infinite in x-direction. Set PACTP = -1. if package is infinite in y direction.

Variable Name	Card Number	Columns	Definition
UUR(N)	3	1-10	Radial velocity of all material in the package. (E10.4)
VVA(N)		11-20	Axial velocity of all material in the package. (E10.4)
SSIEN(N)		21-30	Specific internal energy of all material in the package.(E10.4)
RHOIN(N)		31-40	Initial density of all material in the package. (E10.4)
CZERO(N)	4	1-10	Y_0. Yield strength of all material in package N =
STKI(N)		11-20	Y_1 $\left(Y_0 + Y_1\mu + Y_2\mu^2\right)\left(1 - E/E_0\right)$
STK2(N)		21-30	Y_2 where
STEZ(N)		31-40	E_0 $\mu = \rho/\rho_0 - 1.$ E = specific internal energy.(4E10.4)
RMU(N)		41-50	Rigidity modulus of all material in package N. (E10.4)
AMDM(N)		51-60	Failure criterion based on compression for all material in package N. If compression of material in a cell is less than AMDM, the cell pressure cannot be negative. (Usually AMDM = 0.9 to 0.99). (E10.4)

3. Defining Tracer Particles Along Each Line Segment of the Void Package Boundary. (One card for each line segment. Omit these cards when NSIDES = 0.)

Variable Name	Columns	Definition
STRTX	1-10	x-coordinate (cm) of starting point. If point is at infinity, STRTX = -1.
STRTY	11-20	y-coordinate (cm) of starting point. If point is at infinity, STRTY = -1.
ENDX	21-30	x-coordinate (cm) of end point. If point is at infinity, ENDX = -1.
ENDY	31-40	y-coordinate (cm) of end point. If point is at infinity, ENDY = -1.

(1) SETUP will read "NSIDES" cards, one for each line segment of the free surface.

(2) The input of these tracers should be ordered such that when one travels between consecutive points, the void is on the left.

(3) If the void boundary is spherical the tracers for the void package are automatically defined when the spherical package is generated.

7.2.4 Sample Input Deck for Setting up a HELP Problem

The input for setting up an impact of an iron cylinder onto a one-centimeter aluminum plate is listed below. This impact problem has been run on the version of the code listed in this report and is a suggested test problem for the new user of the HELP code.

```
                        IRON ALUMINUM IMPACT
1   151126.5
       1126.5
2     512.
2     612.
2    15140.
     161.001
     2411.0    -03
     271-1.
2    33131.
2    35162.
2    4211.
     4515.0    -07
2    4717.
2    48130.
2    6715.
2    6912.
     6912.
     7011.
2    7212.
2    731250.
2    781400.
2    8111.
2    8311.
     8611.0    +06
    11011.0    -05
    1131.4
    1371.05
    1391.0    -03
    1441.0    -11
    1491.5
1    5012.0    -06
   20    1    1    1    7.5    -029.6    -029.9    -021.14    -01
      1    1    1    1    1.31   -011.51   -011.73   -011.99    -01
      1    1    1    1    2.29   -012.64   -013.04   -013.50    -01
   999
1
        3        .375        1.40         0.        .25
0.          9.0    +040.          7.8
6.0      +090.          0.          3.0    +101.93    +12.97
1
        4        3.25        2.4         C.        1.4
0.          0.          0.          2.79
3.0      +090.          0.          7.0    +092.74    +11.985
0.          2.4        3.25        2.4
3.25        2.4        3.25        1.4
3.25        1.4        .375        1.4
.375      1.4        .375        .25
.375        .25        0.          .25
1   15010.
```

The lines that follow are part of the printed output from the sample problem described above. This calculation was made on a UNIVAC 1108.

PROBLEM	TIME	CYCLE	TOT.EN.THEOR.	MAX.REL.ERROR-CYCLE	IE SET TO ZERO-PHZ	PLASTIC PLASTIC WORK
26.5000	5.0000900-07	25	1.6049437+10	5.901764A-07 4	-5.9048268+02	1.3417525+08

PACKAGE NO.	IE	KE	TOT.EN. (SUM)	MASS	MU	MV(POSITIVE)	MU	PLASTIC-WORK
1	3.1505310+08	1.4569249+10	1.4904322+10	3.9628230+00	3.3569724+05	3.3569724+05	1.0062024+04	5.7869932+07
2	5.2186947+08	4.2324955+08	1.1451190+09	9.2580599+01	2.0756713+04	2.1420905+04	1.3652366+04	7.2534451+07
TOTALS	8.3692257+08	1.5212518+10	1.6049441+10	9.6543422+01	3.5645395+05	3.5731814+05	2.3714392+04	1.3040436+08

BOUNDARY	BOTTOM	RIGHT	TOP	SEVAPORATED%
MASS OUT	0.0001000	0.0000000	0.0000000	0.0000060
ENERGY OUT	0.0000000	0.0000000	0.0000000	0.0000000
MU OUT	0.0000000	0.0000000	0.0000000	0.0000000
MV OUT	0.0000000	0.0000000	0.0000000	0.0000000
WORK DONE	0.0000000	0.0000000	0.0000000	

TAPE 7 DUMP ON CYCLE   25

CELL-COORDINATES OF TRACERS FOR EACH MATERIAL PACKAGE

PACKAGE 1

N	X	Y	N	X	Y	N	X	Y	N	X	Y	N	X	Y
1	.00	5.90	2	.50	5.90	3	1.00	5.90	4	1.50	5.90	5	2.00	5.90
6	2.50	5.90	7	3.00	5.90	8	3.50	5.90	9	4.00	5.90	10	4.50	5.90
11	5.00	5.90	12	5.00	6.40	13	5.00	6.90	14	5.00	7.40	15	5.00	7.90
16	5.00	8.40	17	5.00	8.90	18	5.00	9.40	19	5.00	9.90	20	5.00	10.40
21	5.00	10.90	22	5.00	11.40	23	5.00	11.90	24	5.00	12.40	25	5.00	12.90
26	5.00	13.40	27	5.00	13.90	28	5.00	14.40	29	5.00	14.90	30	5.00	15.40
31	5.00	15.90	32	5.00	16.40	33	5.00	16.90	34	5.00	17.40	35	5.00	17.90
36	5.00	18.40	37	5.00	18.90	38	5.00	19.40	39	5.00	19.90	40	5.00	20.40
41	5.00	20.90	42	5.00	21.40	43	5.00	21.89	44	5.00	22.39	45	5.00	22.89
46	5.00	23.39	47	5.00	23.88	48	5.00	24.37	49	5.01	24.86	50	5.02	25.35
51	5.04	25.83	52	5.04	26.27	53	5.05	26.71	54	5.11	27.20	55	5.17	27.70
56	5.19	28.10	57	5.20	28.41	58	4.71	28.46	59	4.13	28.52	60	3.53	28.58
61	3.03	28.60	62	2.53	28.62	63	2.02	28.61	64	1.50	28.61	65	1.00	28.61
66	.50	28.61	67	.00	28.61	68	.00	5.90						

PRESSURE

SYMBOL	.	A	B	C	D	E	F	G	H
MAXIMUM VALUE	0.00	4.81+09	9.62+09	1.44+10	1.92+10	2.40+10	2.89+10	3.37+10	3.85+10
SYMBOL	I	J	K	L	M	N	O	P	Q	R
MAXIMUM VALUE	4.33+10	4.81+10	5.29+10	5.77+10	6.25+10	6.73+10	7.21+10	7.69+10	8.17+10	8.66+10
SYMBOL	S	T	U	V	W	X	Y	Z
MAXIMUM VALUE	9.14+10	9.62+10	1.01+11	1.06+11	1.11+11	1.15+11	1.20+11	1.25+11
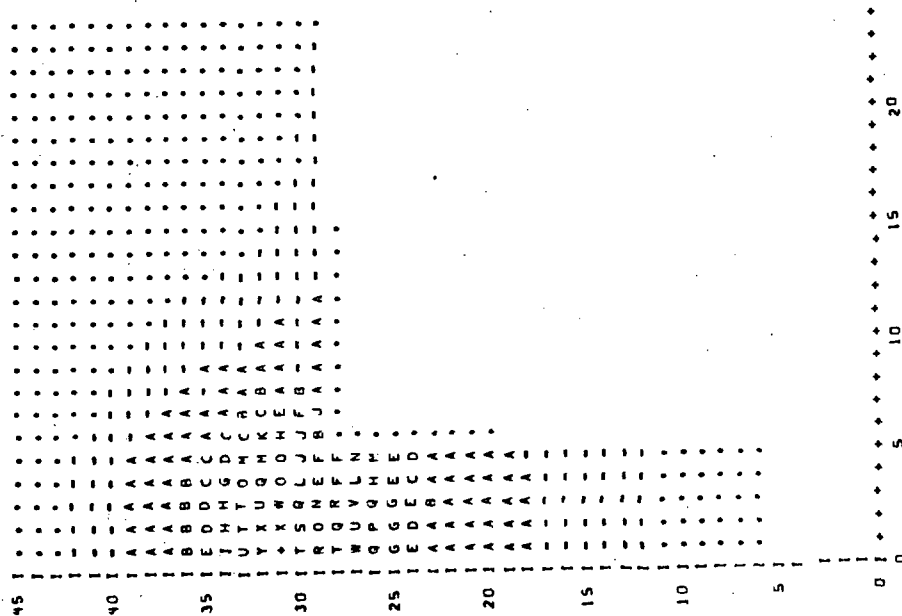
J = 1   R(1) =   .075   DG(1) =   .7500000-01

J	MFLAG	U	V	P	SIE	COMP	TMASS	SZZ	SRR	SRZ	Z
45	2	0.0000	0.0000	0.0000	0.0000	1.00000+00	2.4652-03	0.0000	0.0000	0.0000	2.250+00
44	2	0.0000	0.0000	0.0000	0.0000	1.00000+00	2.4652-03	0.0000	0.0000	0.0000	2.200+00
43	2	6.0191-02	2.0563+00	0.0000	0.0000	1.00000+00	2.4652-03	0.0000	0.0000	0.0000	2.150+00
42	2	-7.3477-02	2.7104+01	2.3091+07	1.5840+02	1.0003+00	2.4652-03	-1.1976+07	5.9958+06	-5.0573+05	2.100+00
41	2	-1.3C03+00	5.9C44+01	1.0941+08	4.3125+03	1.0001+00	2.4655-03	-8.0796+07	4.0057+07	7.2074+05	2.050+00
40	2	-1.0893+00	2.4775+02	2.1292+08	1.8961+04	1.00028+00	2.4659-03	-1.6620+08	8.3477+07	3.3449+05	2.00+00
39	2	1.0037+00	4.8C52+02	7.5191+08	2.2282+05	1.00100+00	2.4676-03	-6.0653+08	3.0254+08	-1.0020+06	1.950+00
38	2	1.1759+00	1.44C8+03	1.1353+09	7.5593+05	1.00177+00	2.4695-03	-1.1185+09	5.6001+08	-3.7495+06	1.90+00
37	2	8.4572+00	2.4113+03	4.9147+08	4.91147+06	1.00441+00	2.4760-03	-3.0592+09	1.5291+09	-4.3586+06	1.850+00
36	2	1.2883+01	5.3945+03	5.0614+09	1.3346+07	1.0076+00	2.4490-03	-3.4584+09	1.7324+09	-2.2998+07	1.800+00
35	2	7.6547+01	9.9306+03	9.4889+09	9.4916+07	1.02459+00	2.5258-03	-3.4215+09	1.7087+09	-1.2244+07	1.750+00
34	2	1.1873-02	2.9606+04	3.9273+11	3.3835+08	1.04760+00	2.5825-03	-3.3118+09	1.6597+09	-4.3951+07	1.700+00
33	2	5.8627+02	4.8911+04	1.0086+11	1.6198+09	1.11004+00	2.7364-03	-2.6800+09	1.3343+09	-4.4573+07	1.650+00
32	2	1.0114+03	7.R200+04	1.1630+11	2.2132+09	1.1228+00	2.7679-03	-2.3682+09	1.1679+09	-1.6097+08	1.600+00
31	2	2.4487+03	2.4487+04	1.2503+11	2.5423+09	1.12997+00	2.7456-03	2.1024+09	-1.2694+09	-4.9939+08	1.550+00
30	2	2.2C6C+03	5.9476+04	9.4822+10	1.5336+09	1.10391+00	2.7213-03	2.6960+09	-1.4011+09	-9.C417+07	1.500+00
29	2	1.4391+03	6.C910+04	8.2715+10	1.1133+09	1.00000	5.5083+00	4.1057+09	-2.0247+09	8.0608+08	1.450+00

MAT 3 4   134

	RHO	SIE	COMP	MASS
3	0.17568+00	1.14661+09	1.04819+00	4.48585-03
4	3.05309+00	9.67032+08	1.09430+00	1.02249-03

FRAC. VOL.   .671   .379   KHO(NVOID,M)= .0

J	MFLAG	U	V	P	SIE	COMP	TMASS	SZZ	SRR	SRZ	Z
28	1	1.1057-03	5.9486+04	9.4C10+10	5.1667+08	1.06347+00	7.3223-03	-4.2792+09	-7.1672+08	3.5892+09	1.400+00
27	1	9.3941+02	6.8193+04	1.C759+11	5.3205+08	1.07278+00	7.3935-03	-6.7754+09	3.1425+09	4.8863+08	1.350+00
26	1	3.7083+02	7.C136+04	7.8018+10	3.1611+08	1.05447+00	7.2672-03	-6.8432+09	3.3045+09	3.1566+08	1.300+00
25	1	1.1434+02	7.6336+04	3.2562+10	6.7276+07	1.0147+00	7.0582-03	-6.9109+09	3.4240+09	1.5560+08	1.250+00
24	1	4.0053+01	8.5944+04	1.9623+10	6.2736+07	1.01428+00	6.9938-03	-6.9212+09	3.4365+09	5.9729+07	1.200+00
23	1	7.3600+00	8.7220+04	4.4656+09	3.5073+06	1.00358+00	6.9268-03	-6.9275+09	3.4261+09	8.2259+07	1.150+00
22	1	1.51A7+01	8.8636+04	3.1761+09	2.0A89+06	1.00245+00	6.9008-03	-6.9275+09	3.4261+09	3.0936+07	1.100+00
21	1	1.51A7+01	8.8753+04	1.7951+09	7.0045+05	1.00139+00	6.9015-03	-6.6501+09	2.2825+09	1.1773+08	1.050+00
20	1	3.8C22+00	8.9147+04	1.4759+09	4.6564+05	1.00115+00	6.8998-03	-5.3986+09	2.7194+09	1.7653+08	1.000+00
19	1	3.8646+00	8.9344+04	9.0067+08	1.6593+05	1.00070+00	6.8967-03	-3.0313+09	1.5741+09	5.2847+07	9.500-01
18	1	-1.6698+00	8.9670+04	6.0320+08	6.9960+04	1.00047+00	6.8951-03	-1.9664+09	9.9763+08	6.7051+07	9.000-01
17	1	-4.2675+00	8.9806+04	2.9112+08	1.8592+04	1.00012+00	6.8934-03	-9.1859+08	4.8115+08	1.3808+07	8.500-01
16	1	-1.3290+00	9.5004+04	1.5004+09	5.7942+03	1.0005+00	6.8927-03	-4.4176+08	2.2409+08	1.3233+07	8.000-01
15	1	-2.0111+00	8.9962+04	6.4701+07	2.4960+03	1.00005+00	6.8922-03	-1.8570+08	9.6883+07	1.9842+06	7.600-01
14	1	-1.6747+01	8.9945+04	2.5604+07	6.9A0+03	1.00002+00	6.8920-03	-5.9762+07	2.9863+07	1.1381+06	7.000-01
13	1	-2.8755+01	8.9979+04	9.6919+06	1.6786+03	1.00001+00	6.8919-03	-1.4746+07	9.5851+06	3.0394+05	6.500-01
12	1	1.227-02	8.9999+04	1.7616+06	1.6640+03	1.00000+00	6.8919-03	0.0000	0.0000	0.0000	6.000-01
11	–	0.0000	9.0000+04	0.0000	1.6640+03	1.00000+00	6.8919-03	0.0000	0.0000	0.0000	5.500-01
10	–	0.0000	9.0000+04	0.0000	1.6640+03	1.00000+00	6.8919-03	0.0000	0.0000	0.0000	5.000-01
9	–	0.0000	9.0000+04	0.0000	2.5600+03	1.00000+00	6.8919-03	0.0000	0.0000	0.0000	4.500-01
8	–	0.0000	9.0000+04	0.0000	3.5599+03	1.00000+00	6.8919-03	0.0000	0.0000	0.0000	4.000-01
7	–	0.0000	9.0000+04	0.0000	3.4890+03	1.00000+00	6.8919-03	0.0000	0.0000	0.0000	3.500-01
6	INI	0.0000	9.0000+04	0.0000							3.000-01

MAT 3

	RHO	SIE	COMP	MASS
3	7.8000+06	3.48800+03	1.0000+00	6.89190-04
	0.00000	0.00000	0.00000	0.00000

FRAC. VOL.   .100   .000   RHO(NVOID,M)=1.0

J	MFLAG	U	V	P	SIE	COMP	TMASS	SZZ	SRR	SRZ	Z
5	0	0.0000	0.0000	0.0000	0.0000	0.00000	0.0000	0.0000	0.0000	0.0000	2.500-01
4	0	0.0000	0.0000	0.0000	0.0000	0.00000	0.0000	0.0000	0.0000	0.0000	2.000-01
3	0	0.0000	0.0000	0.0000	0.0000	0.00000	0.0000	0.0000	0.0000	0.0000	1.500-01
2	0	0.0000	0.0000	0.0000	0.0000	0.00000	0.0000	0.0000	0.0000	0.0000	1.000-01
1	0	0.0000	0.0000	0.0000	0.0000	0.00000	0.0000	0.0000	0.0000	0.0000	5.000-02

7.3 RESTARTING PROBLEMS

During the calculation, HELP periodically writes on a tape or drum file the problem parameters and the current state of the material in each cell. By reading this file and a few input cards, the code can "restart" and continue a calculation from an intermediate point.

The three cards that must be included in a restart deck are as follows:

1. the heading card

2. the "restart" card

3. the last card

The heading card can contain any alphanumeric symbols between columns 2 and 72. The "restart" card defines the four variables described below.

Variable Name	Columns	Definition
PK(1)	8-16	Problem number.
PK(2)	17-25	Restart cycle number.
PK(3)	26-34	Restart flag. = -1. prints long edit of restart cycle. = -2. prints short edit of restart cycle. = -3. restarts from HLPCLM output and prints long edit of restart cycle.
PK(4)	35-43	Number of material packages in the problem (excluding the void package).

This restart card is read by CARDS and columns 1 through 7 are punched as follows

1	2	3	4	5	6	7	(Columns)
1			1	5	1	4	

which indicate there are four words defined by the card, the first one stored in the 151st location of blank common.

The last card defines TSTOP, the time to stop the calculation. This must be the last card of the restart deck even when the problem is stopped on a specified cycle. The TSTOP card is punched as described above for a setup deck, e.g.,

1	2	3	4	5	6	7	8	9	10	12	13	14	15	16	(Columns)
1				5	0	1	1.	0				-	0	6	

Many problem parameters stored in blank common can be redefined when restarting at an intermediate point in the calculation. For example, the number of times PH3 is subcycled, the frequency of printing, or the round-off epsilon can be changed by including in the restart deck cards that redefine these variables. These extra cards must be added between the "restart" card and the last card, i.e., between the two cards that have a "1" in column 1. All cards except the heading card of the restart deck are read by the CARDS subroutine with the format described in Section 7.2.2.

The three cards listed below could be used to restart on cycle 78 the sample problem described in Section 7.2.4. Execution of that problem would continue until T = 3.0 μsec.

IRON ALUMINUM IMPACT

1		151	426.5		78.	-2.	2.
1		50	13.0	-.06			

7.4 DICTIONARY OF IMPORTANT VARIABLES

This section includes a description of the use, units, dimension, and location of all variables in named or blank commons and some variables local to the subroutines. The following conventions are used in the dictionary:

(NAME)   Variable is local to subroutine
            NAME.

NAME   Variable is in common block NAME.

B.C.   The variable is in Blank Common
            or equivalenced to a variable in
            Blank Common.

=Z(N)   The variable is equivalenced to a
            member of the Z-array, the first
            array in Blank Common. These
            variables are usually used in
            setting up and restarting problems.

Variable Name	Location	Number of Words	Units	Definition
AIX	B.C.	2500	ergs/g	Specific internal energy in a cell. (IMAX by JMAX array.)
ALE	(MAP)	41	--	This array has alphabetic characters for positive values in the density, velocity and energy maps. Defined in a DATA statement.
AMDM	B.C.	3	--	INPUT parameter. A material i with compression > $AMDM_i$ is considered solid.
				Usual value: 0.95 to 0.99.
				Used in EQST in testing whether to allow negative pressures (tensions) in pure cells. Used in PH3 to calculate SOLID = AMDM * RHOZ, which determines whether material stresses are nonzero.
AMMP	(PH2)	1	g	Mass transported across right boundary of a cell. (See Appendix B).
AMMU	(PH2)	1	cm-g/sec	Radial momentum transported across the bottom boundary of a cell. (See Appendix B).
AMMV	(PH2)	1	cm-g/sec	Axial momentum transported across the bottom boundary of a cell (See Appendix B).
AMPY	(PH2	1	g	Mass transported across top boundary of a cell. (See Appendix B)
AMUR	(PH2)	1	cm-g/sec	Radial momentum transported across right boundary of a cell. (See Appendix B)
AMUT	(PH2	1	cm-g/sec	Radial momentum transported across top boundary of a cell. (See Appendix B)
AMVR	(PH2)	1	cm-g/sec	Axial momentum transported across right boundary of a cell. (See Appendix B)
AMVT	(PH2)	1	cm-g/sec	Axial momentum transported across top boundary of a cell. (See Appendix B)
AMX	B.C.	2500	g	Total mass in a cell. (IMAX by JMAX array)
BBAR	=Z(149)	1	--	Used in CDT. An INPUT parameter used in local sound-speed calculation for all materials other than a polytropic gas. (Local sound-speed for material i in cell K is approximated as (C_{0_i}) + BBAR · $\sqrt{P(K)}$, where the coefficient B is obtained by determining a typical slope for the isentropes in Ref. 5 and using the relation $C = V\sqrt{-dP/dV}$ to evaluate B at a particular point.

Variable Name	Location	Number of Words	Units	Definition
BBOUND	=Z(74)	1	ergs	Calculated in PH3. Printed in EDIT under "Elastic Plastic Work." Total work done by the elastic and plastic stresses.
BOTM	=Z(39)	1	g	Calculated in PH2. Printed in EDIT. Total mass transported across bottom of grid.
BOTMU	=Z(64)	1	cm-g/sec	Calculated in PH2. Printed in EDIT. Total radial-momentum transported across bottom of grid.
BOTMV	=Z(40)	1	cm-g/sec	Calculated in PH2. Printed in EDIT. Total axial-momentum transported across bottom of grid.
CNAUT	MXCELL	30	cm/sec	Used in CDT. Approximate sound-speed of nineteen materials defined in a DATA Statement in BLOCK. $$C_{Oi} = \sqrt{\frac{ESCAPA_i}{RHOZ_i}} = \sqrt{A_i/\rho_{Oi}}\ .$$
CVIS	=Z(27)	1	--	INPUT parameter. Used to describe the bottom boundary-condition in PH1, PH2 and PH3. Bottom boundary is transmittive when CVIS = -1., reflective when CVIS = 0.
CYC	B.C.	1	--	Defined in INFACE. Used in FLUX to define flux terms of cells that become mixed after first subcycle of INFACE.
CYCLE	=Z(2)	1	--	Used in INPUT, SETUP, CDT, EDIT. Cycle number (an integer value in floating point form).
CYCMX	=Z(69)	1	--	INPUT parameter. Equals number of passes through INFACE on each cycle. Suggested minimum of 2 and maximum of 8. Used to minimize transport noise near interfaces.
CYCPH3	=Z(70)	1	--	INPUT parameter. Number of times to subcycle PH3. If value if -1., PH3 is omitted.
CZERO	B.C.	3	dynes/cm^2	Value of Y_O for yield strength calculation. Defined by input cards for each material package in the grid. Used in STRNG. (See STRENG)
DDX	B.C.	52	cm	An array equivalenced to the DX array such that DDX(1) = DX(0).
DDY	B.C.	102	cm	An array equivalenced to the DY array such that DDY(1) = DY(0).
DELEB	(PH2)	1	ergs	Total energy associated with mass transported across bottom boundary of a cell. (See Appendix B)
DELER	(PH2)	1	ergs	Total energy associated with mass transported across right boundary of a cell. (See Appendix B)

Variable Name	Location	Number of Words	Units	Definition
DELET	(PH2)	1	ergs	Total energy associated with mass transported across top boundary of cell. (See Appendix B)
DELI	(PH2, PH3)	1	ergs/g	Change of specific internal energy of a cell.
DELM	(PH2)	1	g	Total change in mass of a cell.
DELU	(PH2, PH3)	1	cm/sec	Change of radial velocity of a cell.
DELV	(PH2, PH3)	1	cm/sec	Change of axial velocity of a cell.
DMIN	=Z(24)	1	--	INPUT parameter. Allowable relative error in energy sum. When relative error is > DMIN, calculation is terminated. The energy error is computed in EDIT. If everything is working right you should be able to use 10^{-3} for DMIN.
DT	=Z(3)	1	sec	Time step. Calculated in CDT.
DTFACT	(PH3)	1	--	Factor used in calculating a variable time step when subcycling the PH3 calculations.
DTMIN	=Z(144)	1	sec	INPUT parameter. Minimum time step; used in CDT. After STAR = FINAL, if DT < DTMIN execution is stopped.
DTNA	=Z(26)	1	sec	DT from previous time cycle. Used in INPUT, CDT, and EDIT.
DTNOW	(EDIT)	1	sec	Used for saving DT when calling CDT to recalculate pressures after a REZONE.
DTSTR	Local	1	sec	Used in PH3. Time step for a PH3 subcycle.
DX	B.C.	50	cm	The radial-dimension of cells. Equivalenced to DDX such that DDX(1) = DX(0).
DXF	=Z(136)	1	cm	An INPUT parameter used to calculate the DX array if the radial dimension of the cells is uniform.
DY	B.C.	100	cm	The axial-dimension of cells. Equivalenced to DDY so that DDY(1) = DY(0).
DYF	=Z(137)	1	cm	An INPUT parameter used to calculate the DY array if the axial dimension of the cells is uniform.
EAMMP	(PH2)	1	ergs/g	Specific internal energy of mass transported across the right edge of cell.
EAMPY	(PH2)	1	ergs/g	Specific internal energy of mass transported across top of cell.

Variable Name	Location	Number of Words	Units	Definition
ECK	=Z(76)	1	--	Used in EDIT. Relative error in energy sum: $$\left(\sum_k E_k - ETH\right)/ETH;$$ where E_k is total energy in cell K. If $|ECK| >$ DMIN, execution is stopped.
EMIN	=Z(85)	1	ergs/g	INPUT parameter. Minimum specific internal energy of an ideal gas to be used in the pressure iteration in CDT. Usually EMIN = 10^7.
EMOB	=Z(150)	1	ergs	Calculated in PH2. Printed in EDIT. Energy transported across bottom of mesh.
EMOR	=Z(135)	1	ergs	Calculated in PH2. Printed in EDIT. Energy transported across right side of mesh.
EMOT	=Z(146)	1	ergs	Calculated in PH2. Printed in EDIT. Energy transported across top of mesh.
ENERGY	B.C.	1	ergs/g	Defined in CDT as specific internal energy of a material. Used in EQST where $P = f(ENERGY, RHOW)$.
EOB	=Z(134)	1	ergs	Calculated in PH1. Printed in EDIT. Energy change due to work done at bottom boundary of grid.
EOR	=Z(132)	1	ergs	Calculated in PH1. Printed in EDIT. Energy change due to work done at right boundary of grid.
EOT	=Z(133)	1	ergs	Calculated in PH1. Printed in EDIT. Energy change due to work done at top boundary of grid.
ERDUMP	B.C.	1	---	Used in EDIT and ERROR. Flags EDIT to do only a tape dump on an error exit.
ESA	(EQST)	30		Defined in DATA Statement. Values of "a" in equation of state for 19 materials. [=(γ-1)when using perfect gas equation of state.]
ESALPH	(EQST)	30		Defined in DATA Statement. Values of "α" in equation of state for 19 materials.
ESB	(EQST)	30		Defined in DATA Statement. Values of "b" in equation of state for 19 materials.
ESBETA	(EQST)	30		Defined in DATA Statement. Values of "β" in equation of state for 19 materials.
ESCAPA	(EQST)	30	dynes/cm^2	Defined in DATA Statement. Values of "A" in equation of state for 19 materials.

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Variable Name	Location	Number of Words	Units	Definition
ESCAPB	(EQST)	30	dynes/cm^2	Defined in DATA Statement. Values of "B" in equation of state for 19 materials.
ESES	(EQST)	30	ergs/g	Defined in DATA Statement. Values of "E$_s$" in equation of state for 19 materials.
ESESP	(EQST)	30	ergs/g	Defined in DATA Statement. Values of "E$_s'$" in equation of state for 19 materials.
ESEZ	(EQST)	30	ergs/g	Defined in DATA Statement. Values of "E$_0$" in equation of state for 19 materials.
ETH	=Z(13)	1	ergs	Theoretical value of total energy in the mesh. Calculated in SETUP initially; updated in PH1, PH3, PH2 and REZONE.
EVAPEN	=Z(101)	1	ergs	Calculated in PH2 and CDT. Printed in EDIT. Sum of energy lost through "evaporation" of mass left in cells due to round-off error. Adjusted in CDT when "evaporating" isolated cells. Initialized in SETUP.
EVAPM	=Z(100)	1	g	Calculated in PH2 and CDT. Printed in EDIT. Sum of mass lost through "evaporation". See EVAPEN.
EVAPMU	=Z(102)	1	cm-g/sec	Calculated in PH2 and CDT. Printed in EDIT. Sum of radial momenta lost through "evaporation". See EVAPEN.
EVAPMV	=Z(103)	1	cm-g/sec	Calculated in PH2 and CDT. Sum of axial momenta lost through "evaporation". See EVAPEN.
EZPH2	=Z(104)	1	ergs/g	Sum of specific internal energy fluxes that are less than UMIN^2 and are set to zero in PH2. Printed in EDIT.
FINAL	=Z(113)	1	--	INPUT parameter. Maximum value of stability fraction (STAB). If FINAL = 0; the stability fraction will be constant. Used in CDT.
FLEFT	(PH2)	100	cm-g/sec	Radial momentum of mass transported across left side of cell. Equivalenced to UL array. (See Appendix B)
FRACRT	B.C.	1000	cm	The fractional area of the right cell boundary associated with a given material in a mixed cell. Used to compute mass flux of that material.
FRACTP	B.C.	1000	cm	The fractional area of the top cell boundary associated with a given material in a mixed cell. Used to compute mass flux of that material.
GAMC	(PH2)	100	g	Mass transported across left side of cell. Equivalenced to PL array. (See Appendix B)

Variable Name	Location	Number of Words	Units	Definition
GAMMA	=Z(62)	1	--	INPUT parameter. GAMMA = ESA+1. Used in EQST for an ideal gas. User should set GAMMA = 0. when problem does not involve an ideal gas.
I	=Z(88)	1	--	Used in most subroutines as index in radial direction.
ICSTOP	=Z(7)	1	--	INPUT parameter. Used in EDIT. Execution stops on ICSTOP cycle when stopping on a specified cycle rather than time.
GLUED	=Z(79)	1	--	INPUT parameter. GLUED = 1. Causes all mixed cells to be "glued" (see discussion of GLUE subroutine in Section 7.1.10) after the PH1 work terms have been used to update U, V and E. This option is rarely used but can be helpful in stabilizing the interface between a hot gas expanding into a cold dense material.
IEXTX	=Z(123)	1	--	Used in REZONE. When IEXTX = 1., grid is rezoned in x-direction. Enables user to rezone grid in x-direction only.
IMAX	=Z(33)	1	--	INPUT parameter. Number of columns in mesh. IMAX must be an even number if grid is to be rezoned in the x-direction.
IMAXA	=Z(34)	1	--	IMAX + 1.
INTER	=Z(87)	1	--	INPUT parameter. On print cycles if INTER ≠ 0, EDIT will print after CDT, PH1, and PH3. If INTER = 99, in addition to extra EDIT prints, energy total and elastic and plastic work are printed in PH3. If INTER = 7, energy totals are printed in PH2 in addition to the extra EDIT prints.
IPCYCL	=Z(49)	1	--	INPUT parameter. Used in EDIT. The number of cycles between EDIT prints when editing on cycles rather than time.
IPR	=Z(15)	1	--	Maximum number of iterations to be performed by CDT to achieve pressure equilibration between materials in mixed cells.
IVARDX	=Z(83)	1	--	When IVARDX = 1, radial dimension of cells is variable.
IVARDY	=Z(54)	1	--	When IVARDY = 1, the axial dimension of cells is variable.

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Variable Name	Location	Number of Words	Units	Definition
I1	=Z(47)	1	--	INPUT parameter. I1 is used to limit calculation in the radial direction to an "active mesh." Beyond I1 nothing is yet distrubed from the initial conditions. I1 is specified initially as (2 + the column-number of the last column in which there is a non-zero velocity or internal energy). I1 is increased automatically as inactive cells become active. However, I1 is never larger than IMAX.
I2	=Z(48)	1	--	INPUT parameter. Like I1 but for axial disturbance limit.
I3	B.C.	1	--	Used in EDIT as a flag for "short" or "long" prints. I3 = number of columns of grid whose U, V, P, AIX, etc. are listed.
J	=Z(89)	1	--	Used as row-index in most subroutines.
JCENTR	=Z(147)	1	--	Used in GLUE and PH2 to maintain symmetry of calculation involving a sphere. JCENTR = J-line coincident with sphere center.
JEXTY	=Z(124)	1	--	A flag used in REZONE. When JEXTY = 1, grid is rezoned in y-direction.
JMAX	=Z(35)	1	--	INPUT parameter. Number of rows in the grid. JMAX must be an even number if the grid is to be rezoned.
JMAXA	=Z(36)	1	--	Calculated in SETUP, (JMAX + 1).
K	=Z(90)	1	--	Used as cell-index in all subroutines. K = (J-1)*IMAX + I + 1.
KA	B.C.	1	--	Used as cell-index for cell above.
KB	B.C.	1	--	Used as cell-index for cell below.
KMAX	=Z(37)	1	--	Calculated in SETUP (= IMAX*JMAX + 1). K-index of last cell in grid.
KMAXA	=Z(38)	1	--	Calculated in SETUP (KMAX + 1).
KSPACE	(EDIT)	1	--	Used for spacing printed output.
KUNITR	=Z(14)	1	--	Unit number of tape read by INPUT. Must be 7 when reading a "CLAM" tape (i.e., when PK(3) = -3.).
KUNITW	=Z(17)	1	--	Unit number of tape written on by SETUP and EDIT. Set KUNITW ≠ 7 when reading a "CLAM" tape to prevent SETUP from writing over it.
M	=Z(91)	1	--	Usually used as an index for cell K in the mixed cell arrays. M = MFLAG(K)-100.

Variable Name	Location	Number of Words	Units	Definition
MA	B.C.	1	--	Usually value of MFLAG for cell above cell K.
MAPS	=Z(42)	1	--	INPUT parameter that determines the printing of symbolic maps on EDIT cycles. If MAPS = 0, maps are not printed; if MAPS = 1, they are printed.
MAT	B.C.	30	--	Material code numbers for the packages. MAT(1) = 3 indicates the material in package one is iron. See list in Section 2.3 of this report or comments in listing of EQST subroutine.
MAXX	=Z(120)	1	--	Used in ADDTCR. Right column and top row, respectively, of region in which tracer particles are to be added (see NADD, MINX, MINY).
MAXY	=Z(122)	1	--	
MBB	=Z(116)	1	--	Maximum number of coordinates used to describe a package which extends beyond the original grid. (Maximum value in MPAC array).
MBBB	=Z(115)	1	--	Number of material packages set up with finite boundaries extending beyond the limits of the original grid.
MFK	B.C.	1	--	Usually = MFLAG(K).
MFLAG	B.C.	2500	--	Flag for each cell that indicates whether it is pure or mixed. If MFLAG(K) < 100 cell is pure. If MFLAG(K) > 100 cell is mixed and flag gives storage location in mixed cell arrays for cell K (see SIE).
MINX	=Z(119)	1	--	Used in ADDTCR. Left column and bottom row respectively of region in which tracer particles will be added (see NADD, MAXX, MAXY).
MINY	=Z(121)	1	--	
MO	B.C.	1	--	Defined in INFACE. MFLAG(K) of cell K before it becomes mixed. Used in NEWMIX to define mixed cell variables for cell K.
MPAC	B.C.	3	--	MPAC(N) is the number of coordinates used to describe the boundaries of material package N which extends beyond the boundaries of the original grid.
MPACK	B.C.	3	--	MPACK(M) is the package number of the Mth package that extends beyond the boundaries of the original grid.
MR	B.C.	1	--	Value of MFLAG for cell on right.
N	B.C.	1	--	In EQST, N is material code number transferred from CDT.

Variable Name	Location	Number of Words	Units	Definition
N6	=Z(56)	1	--	INPUT parameter. Used in EQST. Negative pressures are allowed in cells above J = N6. N6 is redefined in REZONE. Setting N6 = 0 allows negative pressures everywhere. On the other hand to make sure that negative pressures are always set to zero give N6 a very large value (many times as large as JMAX).
N10	=Z(60)	1	--	Used in CDT to identify I-index of cell which controls the time step.
N11	=Z(61)	1	--	Used in CDT to identify J-index of cell which controls the time step.
NADD	=Z(118)	1	--	INPUT parameter. When NADD > 0 on the first cycle of a run, ADDTCR is called and adds at most NADD material tracer particles between existing tracers in a given region. It will not put more than NTRACR tracers in a cell for each material (see also MINX, MINY, MAXX, MAXY).
NC	=Z(30)	1	--	Cycle number. Set initially to -1 in input. Incremented thereafter in CDT.
NDUMP7	=Z(6)	1	--	INPUT parameter. Used in EDIT to control frequency of tape dumps. E.g., tape dump will occur on every EDIT print when NDUMP7 = 1, or on every fifth EDIT print when NDUMP7 = 5.
NECYCL	=Z(77)	1	--	Defined and printed in EDIT. The cycle during which the largest relative error in the energy sum was computed.
NERR	B.C.	1	--	Used in ERROR as exit flag. Prevents ERROR from being called more than once during a single run.
NFRELP	=Z(5)	1	--	INPUT parameter. Used in EDIT to control frequency of "long" prints. A "long" print will occur with every EDIT if NFRELP = 1; with every fifth EDIT if NFRELP = 5 (see I3).
NK	B.C.	1	--	Tells which statement number of a subroutine is near the call to ERROR.
NMAT	=Z(68)	1	--	INPUT parameter. Equals number of material packages in the problem. [Note: each material package can be made up of several disconnected subpackages. Also package N and N+1 can contain the same material but have different initial conditions (ρ,V,E)].

Variable Name	Location	Number of Words	Units	Definition
NMXCLS	=Z(73)	1	--	INPUT parameter. Equals the maximum number of mixed cells that SETUP or NEWMIX can generate. This number should coincide with the dimension of variables in the MXCELL common block.
NODUMP	=Z(96)	1	--	INPUT parameter. Used in EDIT. When NODUMP = 1, no tape dumps are made except by SETUP on cycle 0. Allows user to restart a problem without writing on the restart tape.
NPRINT	B.C.	1	--	NPRINT = 1 during the cycle on which EDIT prints and checks the energy error.
NR	B.C.	1	--	Identifies which subroutine calls ERROR. Used in ERROR for printing error message.
NRC	=Z(32)	1	--	Used in PH1 and PH2 in advancing active grid.
NREZ	=Z(20)	1	--	Defined in SETUP. Equals maximum number of rezones allowed.
NTCC	=Z(81)	1	--	INPUT parameter. When NTCC = 1 passive cell centered tracers are used.
NTRACR	=Z(72)	1	--	INPUT parameter. Used to determine density of material tracer particles. If NTRACR = 2, SETUP will place two tracers per cell edge around a material package.
NSIDES	=Z(67)	1	--	INPUT parameter. Equals number of line segments — other than grid lines — that bound the void. If there is no void in the problem, NSIDES = 0. Used in SETUP to read in cards for defining the tracer particles that circumscribe the void package.
NTPMX	=Z(78)	1	--	INPUT parameter. Equals the maximum number of tracers that SETUP or ADDTCR can generate to circumscribe a single material package. The value of this number should coincide with the dimension of the TX and TY arrays.
NUMREZ	=Z(12)	1	--	INPUT parameter. Initially equals number of rezones allowed in one run. Diminished by one after each rezone. Used in EDIT and REZONE.
NUMSCA	=Z(43)	1	--	INPUT parameter. Number of times the print interval is to be rescaled. Used in EDIT. See PRDELT for further details.

Variable Name	Location	Number of Words	Units	Definition
NUMSP	=Z(4)	1	--	Used in EDIT to count the number of prints (short or long) since the last tape dump.
NVOID	B.C.	1	--	Defined in INPUT: = NMAT + 1, number of material packages + 1. The number of the void package if there is one.
NZ	=Z(19)	1	--	Defined and used in EDIT for 1-D problems. NZ = 4**NRZ. After re-zoning the grid NZ is used to scale the values printed by EDIT for the total mass, energy and momentum.
P	B.C.	2500	dynes/cm^2	Cell-pressure. (IMAX by JMAX array). Calculated in EQST. Used by PH1. [Note: the P-storage space is used for other information in INFACE, PH3 and PH2.]
PACX	B.C.	30	cell	PACX(N,M) is the x-coordinate of the Mth tracer of the Nth package which has boundaries extending beyond the original grid.
PACY	B.C.	30	cell	PACY(N,M) is the y-coordinate of the Mth tracer of the Nth package which has boundaries extending beyond the original grid.
PL	B.C.	200	dynes/cm^2	Used in PH1 for saving pressures on left side of cell. (Storage used for other purposes in CDT and PH2.)
PLW	MXCELL	3	ergs	An array that stores the total work done by plastic stresses in each material package. Calculated in PH3; printed in EDIT.
PMIN	=Z(86)	1	dynes/cm^2	Used as a pressure cut-off. Calculated and printed in CDT: PMIN = min (PMIN, C^2 ρ_0 UMIN). For cycle 0, PMIN defined by an input card.
PRCNT	=Z(16)	1	--	Convergence requirement for equili-brating pressures in a mixed cell. If $\left(\dfrac{P_i - \bar{P}}{\bar{P}}\right) \leq$ PRCNT for all materials (i) in cell K, materials in cell are considered to be pressure equilibrated and P(K) = $\bar{P}$.
PRDELT	=Z(45)	1	sec	INPUT parameter. Gives the initial time interval between EDIT prints. There are five parameters which con-trol printing frequency: PRDELT, IPCYCL, PRLIM, PRFACT, and NUMSCA. If the user is printing on time (PRDELT $\neq$ 0 and IPCYCL = 0), DT will be adjusted so that a print will occur exactly every PRDELT seconds. If the user is printing on cycles (PRDELT = 0, IPCYCL $\neq$ 0), a print will occur every IPCYCL cycles.

Variable Name	Location	Number of Words	Units	Definition
PRDELT (continued)				PRLIM, PRFACT and NUMSCA are used to increase the print interval. PRLIM is the time (or cycle) at which PRDELT (or IPCYCL) and PRLIM are multiplied by PRFACT. The new value of PRLIM establishes the next time (or cycle) when the print interval will be re-scaled. This process continues at most NUMSCA times. EXAMPLE: You wish to print every 1×10^{-8} sec until you reach 1×10^{-7} sec, then every 1×10^{-7} sec until 1×10^{-6} sec and every 1×10^{-6} sec thereafter: PRDELT = $1. \times 10^{-8}$ PRLIM = $1. \times 10^{-7}$ PRFACT = 10. NUMSCA = 2.
PRESUR	B.C.	1	dynes/cm^2	Defined in EQST: pressure = $f(\rho, E)$. Used in CDT to define $P(K)$ in the case of pure cells, and in the case of mixed cells to define the pressure of a material.
PRFACT	=Z(46)	1	--	INPUT parameter. Used in EDIT as a factor for rescaling PRDELT, IPCYCL, and PRLIM when PRLIM-time or cycle is reached (see PRDELT). Should be > 1.
PRLIM	=Z(44)	1.	--	INPUT parameter: time or cycle at which to rescale PRDELT (or IPCYCL) and PRLIM by PRFACT (see PRDELT).
PROB	=Z(1)	1	--	INPUT parameter. Identifying problem number. Should be between 0 and 100.
PROP	B.C.	50	--	Calculated and used in EDIT. For 1-D problems the totals for energy, mass momentum per unit area are computed by dividing by 4** (number of rezones). These totals are stored in PROP for printing. Equivalenced to UL array.
PRR	(PH1)	1	dynes/cm^2	Pressure at right cell boundary.
PRTIME	=Z(131)	1	sec	Initially set to PRDELT in INPUT. Thereafter calculated in EDIT. When T = PRTIME, it is time to print.
RADIUS	=Z(148)	1	cm	INPUT parameter. Radius of sphere (in cms).
RATIO	(CDT)	1	sec	Used in calculation of DT: ratio of $(DX, DY)_{min}$ to $[(U,V)_{max}$ + local sound speed$]_{max}$ for a given cell.
RC	(PH1)	1	cm	Distance from axis to center of a cell.
RELERR	(EDIT)	1	--	Used for storing and printing maximum relative error in the energy sum.

Variable Name	Location	Number of Words	Units	Definition
REZ	=Z(95)	1	--	Flag defined in PH2 and used in EDIT. Signals when the REZONE subroutine should be called. The rezone flag is turned on when material begins to flow out through transmittive boundaries (see VT). If the grid is rezoned in both directions, each set of four cells in the mesh is made into one cell. The new mass is the sum of masses in the four original cells. Momentum and total energy are conserved but in so doing some kinetic energy is changed to internal. (The result is that rezoning has a stabilizing influence.) When all rezones have been performed, or if no rezones are permitted, material is allowed to flow out through transmittive boundaries and the mass and energy are lost from the system.
REZFCT	=Z(71)	1	--	INPUT flag for rezoning. If = 1., the grid is rezoned (NUMREZ) times. If = 0., no rezoning is done. Tested in PH2 and EDIT.
RHOIN	B.C.	3	g/cm^3	The initial densities of material in the packages. Defined by input cards read by SETUP.
RHOW	B.C.	1	g/cm^3	Density of material. Defined in CDT and used in EQST to define pressure: P = f(ENERGY,RHOW).
RHOZ	MXCELL	30	g/cm^3	Defined in DATA statement in BLOCK. Normal density for 19 materials.
RMU	B.C.	3	$dynes/cm^2$	Rigidity modulus values for material packages. Defined by input cards read by SETUP. Used in PH3.
ROEPS	=Z(110)	1	--	INPUT parameter. A round-off epsilon used in calculating cutoffs of energy, velocity, pressure and mass flux.
RR	(PH1)	1	cm	Distance from axis to center of cell on the right.
RTM	=Z(57)	1	g	Calculated in PH2. Printed in EDIT. Total mass lost out right side of grid.
RTMU	=Z(10)	1	cm-g/sec	Calculated in PH2. Printed in EDIT. Total radial-momentum lost out right side of grid.
RTMV	=Z(58)	1	cm-g/sec	Calculated in PH2. Printed in EDIT. Total axial-momentum lost out right side of grid.
SAMMP	MXCELL	750	g	The flux of materials across right boundary of mixed cells. Calculated in FLUX; used in PH2 and INFACE.
SAMPY	MXCELL	750	g	The flux of materials across the top boundary of mixed cells. Calculated in FLUX; used in PH2 and INFACE.

Variable Name	Location	Number of Words	Units	Definition
SDT	B.C.	1	sec	Defined in INFACE = DT/CYCMX. Time step for each subcycle of INFACE.
SIE	MXCELL	750	ergs/g	The specific internal energy of materials in mixed cells. If cell K is mixed and M = MFLAG(K)-100, then SIE(1,M) is the specific internal energy of material one in cell K (see XMASS, RHO).
SIENEW	(PH2)	1	ergs/g	New value of specific internal energy.
SIGC	B.C.	100	ergs	Used in PH2 for internal energy of mass transported across left side of cell. Equivalenced to PL(103). (See Appendix B)
SIGMU	(PH2)	1	cm-g/sec	Total change in radial momentum of a cell.
SIGMV	(PH2)	1	cm-g/sec	Total change in axial momentum of a cell.
SNB	B.C.	50	dynes/cm^2	Normal deviator stress at bottom of a cell. Equivalenced to P array.
SNL	(PH3)	1	dynes/cm^2	Normal deviator stress at left of a cell.
SNLX	(PH3)	1	dynes/cm^2	[= SNL*X(I-1)].
SNR	(PH3)	1	dynes/cm^2	Normal deviator stress at right of a cell. (=S_{11} at right).
SNT	(PH3)	1	dynes/cm^2	Normal deviator stress at top of a cell (=S_{22} at top).
SOLID	(STRNG)	1	g/cm^3	= RHOZ$_i$ * AMDM$_i$. In PH3, if RHO$_i$ < SOLID for one material, i, in cell K, stresses of cell K are set to zero.
SRATIO	(CDT)	1	sec	Used to calculate DT. The smallest ratio in grid of a cell's minimum dimension to the sum of its maximum velocity and sound speed.
SS1	= Z(127)	1	g/ergs	Used in EQST: in the expanded equation of state: = $1/(E'_s - E_s)$.
SS2	= Z(128)	1	--	Constant controlling reflective (and axis) boundary treatment. Defined in INPUT: SS2 = 1. Used in PH2.
SS4	= Z(130)	1	--	INPUT parameter. If SS4 ≠ 0., REZONE is called on first cycle of a run. Used in EDIT.
SSIEN	B.C.	3	ergs/g	Initial specific internal energy of each package. Defined by input cards read in SETUP.
STAB	= Z(139)	1	--	INPUT parameter. Used in CDT. Initial value of "stability fraction" for the calculation of DT. If FINAL = 0., STAB is constant. Otherwise its value progresses from STAB to FINAL in a geometric progression. [Note: DT = STAB * SRATIO]

Variable Name	Location	Number of Words	Units	Definition
STB	(PH3)	1	dynes/cm^2	Shear stress at bottom of cell.
STEZ	B.C.	3	ergs/g	E_0 for each material package. Used in yield-strength calculation in STRNG. Defined by input cards read by SETUP. (See STRENG)
STK1	B.C.	3	dynes/cm^2	Y_1 for each material package. Used in yield-strength calculation in STRNG. Defined by input cards read by SETUP. (See STRENG)
STK2	B.C.	3	dynes/cm^2	Y_2 for each material package. Used in yield-strength calculation in STRNG. Defined by input cards read by SETUP. (See STRENG)
STL	(PH3)	1	dynes/cm^2	Shear stress at left of cell.
STLX	(PH3)	1	dynes/cm	= STL * X(I-1).
STR	(PH3)	1	dynes/cm^2	Shear stress at right of cell.
STRENG	(STRNG)	1	dynes/cm^2	Yield strength of material: STRENG = $(Y_0 + Y_1\mu + Y_2\mu^2) \cdot (1 - E/E_0)$. $\mu = \rho/\rho_0 - 1$. If STRENG $\leq$ 0., stresses are set to 0. If E > E_0, STRENG = 0 and stresses are set to 0. Y_0 is CZERO, Y_1 is STK1, Y_2 is STK2, E is specific internal energy of material, E_0 is STEZ, ρ is density of material, ρ_0 is normal density of material.
STRSRR	ELPL	2500	dynes/cm^2	The cell centered deviatoric normal stress in the radial direction (IMAX by JMAX array).
STRSRZ	ELPL	2500	dynes/cm^2	The cell centered deviatoric shear stress in the radial direction (IMAX by JMAX array).
STRSZZ	ELPL	2500	dynes/cm^2	The cell centered deviatoric normal stress in the axial direction (IMAX by JMAX array).
STT	(PH3)	1	dynes/cm^2	Shear stress at top of cell.
SUM	B.C.	1	--	Used in most routines as working storage when summing quantities.
SUME	(PH2)	1	ergs	Used in PH2. Sums energy fluxes ignored during one cycle. Used to adjust ETH, the theoretical energy total.
T	= Z(84)	1	sec.	Time. Initially defined in INPUT. Incremented in CDT. Adjusted in EDIT for printing and stopping at specified time values.

Variable Name	Location	Number of Words	Units	Definition
TAU	B.C.	50	cm^2	Initially defined in SETUP and redefined in REZONE. Area of cell face: $= \pi[X(I)^2 - X(I-1)^2]$. Used in most subroutines.
TAUDTS	(PH1)	1	cm^2/sec	$= TAU * DT$.
THO3	(PH3)	1	1/sec	$= 1/3\left(u_r + v_z + \frac{u}{r}\right)$.
TNOW	(EDIT)	1	sec	Used in EDIT: = time now; saved when EDIT calls CDT after calling REZONE.
TOPM	= Z(63)	1	g	Calculated in PH2. Printed in EDIT. Total mass transported across top of grid.
TOPMU	= Z(9)	1	cm-g/sec	Calculated in PH2. Printed in EDIT. Total radial-momentum transported across top of grid.
TOPMV	= Z(66)	1	cm-g/sec	Calculated in PH2. Printed in EDIT. Total axial-momentum transported across top of grid.
TRIAL	Local	1	cm/sec	Used in CDT. Maximum sum in the entire grid of a cell's sound-speed and largest velocity component. Printed in CDT as MAXCUV.
TSTOP	= Z(50)	1	sec	INPUT parameter. Value of T at which execution stops when stopping on time rather than cycles.
TWOPI	B.C.	1	--	Calculated in INPUT. Used in PH3: $= 2\pi$.
TX	B.C.	1600	cell	X-coordinates of tracer particles circumscribing the material packages.
TY	B.C.	1600	cell	Y-coordinates of tracer particles circumscribing the material packages.
U	B.C.	2500	cm/sec	Radial velocity of cell (IMAX by JMAX array).
UAMMP	(PH2)	1	cm/sec	Radial velocity of mass transported across right cell edge.
UAMPY	(PH2)	1	cm/sec	Radial velocity of mass transported across top cell edge.
UEFF	(INFACE)	1	cm/sec	Radial velocity of a tracer particle, computed from an area-weighted averaging scheme.
UK	B.C.	150	cm/sec	Temporary storage for three rows of radial velocities.
UKT	(PH3)	1	cm/sec	Temporary storage for U(K) when computing elastic-plastic work done on cell K.
UL	(PH1)	200	cm/sec	Radial velocity at left boundary of cell K. UL array equivalenced to the FLEFT and YAMC arrays used in PH2.
UMIN	= Z(129)	1	cm/sec	Calculated in CDT. Used as velocity cutoff in PH2 $= (ROEPS) * (\text{sound-speed} + \text{velocity})_{max}$.

Variable Name	Location	Number of Words	Units	Description
UNxxx	= Z(xxx)	1	--	Unused Z-storage.
UNEW	(PH2)	1	cm/sec	New radial velocity of cell K.
UOX	(PH3)	1	1/sec	$=2U_K/(X_i + X_{i-1})$. Used to define strain rates of cell K.
URR	B.C.	1	cm/sec	Used in PH1, FLUX and PH2. Temporary storage for radial transport velocity at right boundary of cell K.
UUR	B.C.	3	cm/sec	Initial radial velocities of packages. Defined by input cards read in SETUP.
UVMAX	= Z(80)	1	cm/sec	Calculated in CDT. Maximum velocity in grid. Redefined in PH1: $\max\left(\text{UVMAX}, \sqrt{E_{max}}\right)$. Used to flag cells that should be glued after PH1.
V	B.C.	2500	cm/sec	Axial velocity of cell (IMAX by JMAX array).
VABOVE	B.C.	1	cm/sec	Used in PH1, FLUX, and PH2. Temporary storage for axial transport velocity at top boundary of cell.
VAMMP	(PH2)	1	cm/sec	Axial velocity of mass transported across right cell edge.
VAMPY	(PH2)	1	cm/sec	Axial velocity of mass moving across top cell edge.
VBLO	(PH1)	1	cm/sec	Axial velocity at bottom boundary of cell.
VEFF	(INFACE)	1	cm/sec	Axial velocity of a tracer particle, computed from an area-weighted averaging scheme.
VEL	(PH1)	1	--	Subcycle flag: VEL = 1. on first pass through PH1, and U, V, E are updated. On second pass VEL = 0. and only E is updated using new U,V from first pass.
VK	B.C.	150	cm/sec	Temporary storage for three rows of axial velocities.
VKT	(PH3)	1	cm/sec	Temporary storage for V(K) when computing elastic-plastic work done on cell K.
VNEW	(PH2)	1	cm/sec	New axial velocity of cell K.
VOLSPH	(SETUP)	1	cm^3	Volume of that part of the toroid generated by a cell which is inside the sphere.
VOW	(EQST)	1	--	$= \rho_0/\rho$.

Variable Name	Location	Number of Words	Units	Definition
VT	= Z(55)	1	g/cm^2	Used in PH2 to trigger a rezone of the grid. If mass is transported across grid boundary and if minimum density of mass flux (when spread over a cell) $\geq$ VT, then rezone flag (REZ) is set to one. Defined in INPUT: VT = 10^{-5}.
VVA	B.C.	3	cm/sec	Initial axial velocities of packages. Defined by input cards read by SETUP.
WFLAGF	= Z(51)	1	--	Used in INPUT and EDIT. Set = 1. on first cycle of run in INPUT. Triggers an EDIT print on first cycle of every run. Reset to 0. at end of EDIT.
WFLAGL	= Z(52)	1	--	Used in MAIN and EDIT. Flags last cycle. Set = 1. in EDIT. Signals HELP to call exit.
WS	B.C.			
WSA	B.C.			
WSB	B.C.			
WSC	B.C.	1	--	Used in most subroutines for local working storage.
WSX	B.C.			
WSY	B.C.			
X	B.C.	50	cm	Distance from axis to outside of cell. Equivalenced to XX array such that X(0) = XX(1).
XIENRG	= Z(140)	1	ergs	Total internal energy. Calculated in EDIT and used for printing labels on tracer particle plots.
XKENRG	= Z(141)	1	ergs	Total kinetic energy. Calculated in EDIT and used for printing labels on tracer particle plots.
XMASS	MXCELL	750	g	The mass of materials in mixed cells (see SIE, RHO).
XMAX	= Z(18)	1	cm	= X(IMAX).
XP	TRACRS	1000	cell	X-coordinates of passive cell-centered tracer particles.
XTENRG	= Z(142)	1	ergs	Total energy. Calculated in EDIT and used for printing labels on tracer particle plots.
XUM	(MAP)	41	--	Used in MAP. Array has negative alphabetic characters for maps. Defined in a DATA statement.
XX	B.C.	52	cm	Equivalenced to X array so as to make X(0) available.
Y	B.C.	100	cm	Distance from bottom of grid to top of cell. Equivalenced to YY array such that Y(0) = YY(1).

Variable Name	Location	Number of Words	Units	Definition
YAMC	B.C.	100	cm-g/sec	Calculated and used in PH2. Axial momentum of mass transported across left side of cell. Equivalenced to UL array. (See Appendix B)
YMAX	(SETUP)	1	cm	= Y(JMAX).
YP	TRACRS	1000	cell	Y-coordinates of passive cell-centered tracer particles.
YY	B.C.	102	cm	Equivalenced to Y array so as to make Y(0) available.
Z	B.C.	1	--	Storage for most of the input parameters follows Z, the first work of blank common. The "Z-array" (the first 150 words of blank common) is written on tape for restarting problems (see Appendix A).

VIII. REFERENCES

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APPENDIX A

Z-BLOCK VARIABLES LISTED NUMERICALLY

See Dictionary for meaning and use.

*1.	PROB	23.	UN23	*45.	PRDELT
2.	CYCLE	*24.	DMIN	*46.	PRFACT
3.	DT	25.	UN25	2*47.	I1
4.	NUMSP	26.	DTNA	2*48.	I2
2*5.	NFRELP	*27.	CVIS	2*49.	IPCYCL
2*6.	NDUMP7	28.	UN28	1*R50.	TSTOP
2*7.	ICSTOP	29.	UN29	51.	WFLAGF
8.	PIDY	30.	NC	52.	WFLAGL
9.	TOPMU	31.	UN31	53.	UN53
10.	RTMU	32.	NRC	2*54.	IVARDY
11.	UN11	2*33.	IMAX	55.	VT
2*12.	NUMREZ	34.	IMAXA	2*56.	N6
13.	ETH	2*35.	JMAX	57.	RTM
2*14.	KUNITR	36.	JMAXA	58.	RTMV
2*15.	IPR	37.	KMAX	59.	UN59
*16.	PRCNT	38.	KMAXA	60.	N10
2*17.	KUNITW	39.	BOTM	61.	N11
18.	XMAX	40.	BOTMV	*62.	GAMMA
19.	NZ	41.	NUMSPT	63.	TOPM
20.	NREZ	2*42.	MAPS	64.	BOTMU
21.	IGM	2*43.	NUMSCA	65.	UN65
22.	UN22	*44.	PRLIM	66.	TOPMV

2*67.	NSIDES	93.	UN93	2*119.	MINX
2*68.	NMAT	94.	UN94	2*120.	MAXX
*69.	CYCMX	95.	REZ	2*121.	MINY
*70.	CYCPH3	2*96.	NODUMP	2*122.	MAXY
*71.	REZFCT	97.	UN97	2*123.	IEXTX
2*72.	NTRACR	98.	UN98	2*124.	JEXTY
2*73.	NMXCLS	99.	UN99	125.	UN125
74.	BBOUND	100.	EVAPM	126.	UN126
75.	UN75	101.	EVAPEN	127.	SS1
76.	ECK	102.	EVAPMU	128.	SS2
77.	NECYCL	103.	EVAPMV	129.	UMIN
2*78.	NTPMX	104.	EZPH2	*130.	SS4
*79.	GLUED	105.	UN105	131.	PRTIME
80.	UVMAX	106.	UN106	132.	EOR
2*81.	NTCC	107.	UN107	133.	EOT
82.	UN82	108.	UN108	134.	EOB
2*83.	IVARDX	109.	UN109	135.	EMOR
84.	T	*110.	ROEPS	*136.	DXF
85.	EMIN	111.	UN111	*137.	DYF
*86.	PMIN	112.	UN112	138.	UN138
2*87.	INTER	*113.	FINAL	*139.	STAB
88.	I	114.	UN114	140.	XIENRG
89.	J	115.	MBBB	141.	XKENRG
90.	K	116.	MBB	142.	XTENRG
91.	M	117.	UN117	143.	UN143
92.	N	2*118.	NADD	*144.	DTMIN

145.   UN145

146.   EMOT

2*147.   JCENTR

*148.   RADIUS

*149.   BBAR

1*150.   EMOB = 0   [Last card of SETUP input deck. Not
                     included when restarting from tape.]

1*R151.   PK(1)   should be the same as PROB, problem number.

*R152.   PK(2), the restart cycle.

*R153.   PK(3): when = -1., program will restart from tape
                and do a "long" EDIT prints of the pickup
                cycle. When = -2., program will restart from
                tape and do a "short" EDIT print of the pick-
                up cycle. When = -3., program expects to
                restart from a HLPCLM tape. Do not include
                in setup input deck.

---

*User-supplied input-values.

1*Must have a "1" in column 1.

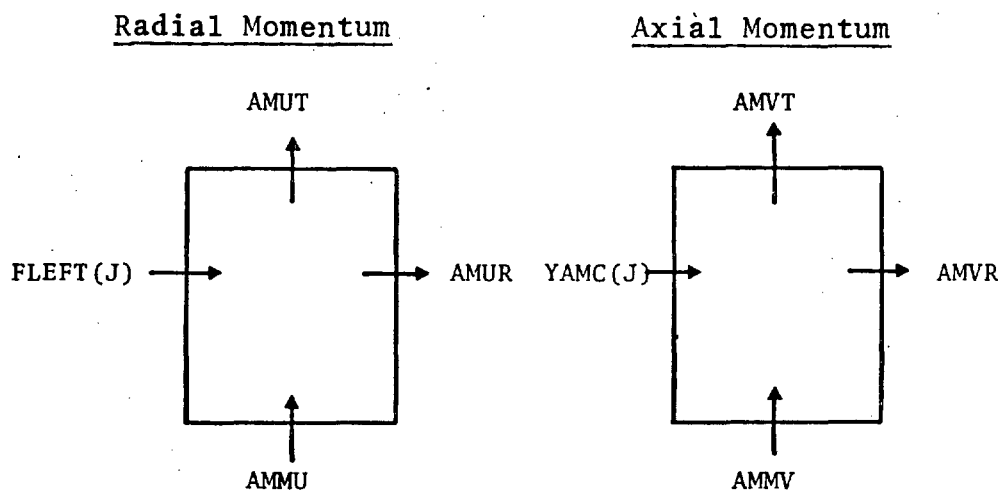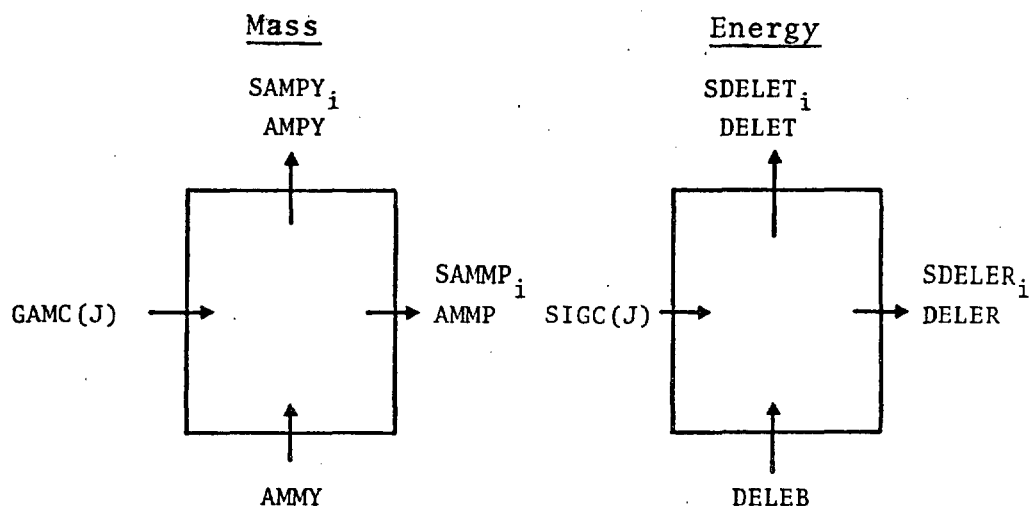2*Must have a "2" in column 1.

RMust be included in a restart input deck.

APPENDIX B
VARIABLES USED IN TRANSPORT ACROSS CELL BOUNDARIES

Mass

$SAMPY_i$
AMPY

GAMC(J) →   → $SAMMP_i$
AMMP

AMMY

Energy

$SDELET_i$
DELET

SIGC(J) →   → $SDELER_i$
DELER

DELEB

Radial Momentum

AMUT

FLEFT(J) →   → AMUR

AMMU

Axial Momentum

AMVT

YAMC(J) →   → AMVR

AMMV

APPENDIX C
FLAGS AND CONVENTIONS GOVERNING INTERFACE CELLS

Statements	Meaning
RHO(1,M) = -1.0	The M location in mixed cell arrays (XMASS, RHO, SIE, FRACTP, FRACRT, SAMMP, SAMPY) is not in use.
NVOID = NMAT + 1	NVOID, the number of the "void" package, is one more than the number of material packages.
M = MFLAG(K)-100 RHO(NVOID,M) = 1.0	Cell K contains the free surface interface.
MFLAG(K) = 0	Cell K is empty and is not cut by an interface.
0 < MFLAG(K) < 100	Cell K is nonempty and pure, i.e., does not contain an interface.
MFLAG(K) = 2	Cell K is completely inside package 2 boundary and contains only material of package 2.
MFLAG(K) > 100	Cell K is mixed, i.e., contains at least one interface. M = FLAG(K)-100 gives location of quantities in mixed cell arrays for cell K.
MFLAG(K) < 0	Cell K was mixed on previous cycle but is no longer cut by an interface and will be reflagged at the end of PH2.
MAT(2) = 3	The material code number of package 2 is 3. The list in EQST indicates that code number 3 corresponds to iron.
MAT(1) = 20	The material of package 1 is an ideal gas which is given special treatment in the sound speed calculation and in the strength phase of the code.
M = MFLAG(K)-100 RHO(N,M) = 0.	The interface of package N does not cut cell K. Material of package N will not be transported into cell K.

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Statements	Meaning
M = MFLAG(K)-100 RHO(N,M) > 0. XMASS(N,M) = 0.	Package N interface cuts cell K, but cell K does not yet contain any material of package N.
M = MFLAG(K)-100 RHO(N,M) = 0. XMASS(N,M) > 0.	Package N interface has left cell K, and the material of package N [XMASS(N,M)] should be completely evacuated on this cycle.

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13. ABSTRACT

Volume I details a new numerical model for solving multi-material flows which are functions of two space dimensions and time. The program, called HELP, is basically Eulerian but also contains Lagrangian features for the explicit definition of interface positions. The program is general in the sense that any number of materials can be present in a given Eulerian cell and no special difficulties arise for flows involving extreme material distortions.

The program is suitable for compressible media with strength dependence. Compressibility is included by an equation of state giving pressure as a function of density and specific internal energy. Strength is included by means of a constitutive equation giving the deviatoric stresses as functions of the elastic and plastic strains.

In this Volume I report, the basic equations are developed, the computer program is documented and sample results are given from six applications.

KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Numerical Finite-Difference Code						
Eulerian Computer Code						
Compressible Flow						
Elastic-Plastic Flow						
Hydrodynamic Code						